Taming the Boundaries

A brief introduction to Heterotic M-Theory

John Omotani

University of Nottingham

14th May 2009

Outline

Outline

String Theories

- 5 consistent (super-)string theories
  - I, IIA, IIB, $SO(32)$ heterotic and $E_8 \times E_8$ heterotic
- Inter-related by dualities
  - eg.

$$\text{Type IIA compactified on circle of radius } R \quad \overset{T}{\longleftrightarrow} \quad \text{Type IIB compactified on circle of radius } \frac{\alpha'}{R}$$

- All part of one theory?

M-Theory

- Full M-Theory is unknown
- Low energy limit is 11d supergravity[1]

$$S = \frac{1}{2\kappa^2}\int\left(dvR - \frac{1}{2}G\wedge *G - \frac{1}{6}C\wedge G\wedge G + (\text{fermions})\right)$$

- Compactification on a circle gives IIA supergravity
- Compactification on an interval gives heterotic supergravity (of which more later)

[1] E. Cremmer, B. Julia, J. Scherk, Phys. Lett. B 76 (1978) 409

11d Content

- Field content is $g_{\mu\nu}$, $C_{\mu\nu\rho}$, ψ_μ
- But also 6-potential with field strength dual to $G = dC$
- p-potential couples to p-dimensional world volume
- $M2$ and $M5$ branes appear as sources for $C_{(3)}$, $C_{(6)}$

10d Content

- Compactifying gives the basic objects of the 10d string theory
  - $M2$ wrapping the circle $\longrightarrow$ string
  - $M2$ not wrapping the circle $\longrightarrow$ $D2$ brane
  - etc.

Aside on Anomalies

- Quantum anomalies[2]: quantum theory fails to respect classical symmetry
  - gauge symmetry: gauge anomaly ($2k$ dimensions)
  - general covariance: gravity anomaly ($4k + 2$ dimensions)
- ie. Quantum effective action varies under gauge transformations/diffeomorphisms
- Perturbatively, shows up in hexagon diagrams

[2] L. Alvarez-Gaumé, P. Ginsparg, Ann. Phys. 161 (1985) 423, erratum–ibid 171 (1986) 233

Outline

Gravity Anomaly

- 11d supergravity on an interval with two 10d boundaries
- 11d supergravity is anomaly free
- But chiral gravitino on the 10d boundary gives gravity anomaly
- Cancel with E_8 vector multiplet on the boundary

Boundary Theory

- Gauge theory on the boundary:
  - Gauge field A_μ with field strength $F_{\mu\nu}$
  - Gaugino χ
- Supersymmetry of boundary determines coupling to bulk supergravity
  - In particular $G \sim F^2$ and so C acquires an E_8 gauge variation

Gauge Anomaly

- 10d super-Yang-Mills also has a gauge anomaly
- Cancelled by (classical) variation of $\int C \wedge G \wedge G$
  - fixes the gauge coupling
- The only parameter in the theory is the gravitational coupling $2\kappa^2$

Outline

What's the Difference?

- Supergravity action should include boundary terms involving the extrinsic curvature, $K_{\mu\nu}$
  - These were neglected by Hořava and Witten
- Non-trivial boundary condition: $K_{\mu\nu} = \kappa^2 \left(T_{\mu\nu} - \frac{1}{9} g_{\mu\nu} T \right)$
- Super-Yang-Mills on the boundary means $T_{\mu\nu} \neq 0$ generally
- Affects the construction of the boundary conditions

Original Formulation

- Hořava and Witten[3]:
  - modified Bianchi identity ($dG = \ldots$) involving δ-functions
  - modified supersymmetry transformations, also involving δ-functions
- Combined effect introduced divergences into the action (terms proportional to $\delta(0)$)
- Theory breaks down entirely beyond order κ^2

[3] P. Hořava, E. Witten, Nucl. Phys. B475 (1996) 94, hep-th/9603142

Improved Formulation

- Moss[4]:
  - Taking into account $K_{\mu\nu}$ leads to modification of the chirality condition of ψ_μ: $P_+\psi_\mu \sim F\chi$
  - supersymmetry transformations are unaltered
  - no δ-functions
- These are physically distinct theories

[4] I. Moss, Nucl. Phys. B729 (2005) 179, hep-th/0403106

Outline

Current Work

- Working on 5d reduction of Moss's improved theory
- Are there phenomenologically important differences from work[5] based on the Hořava-Witten version?

[5] eg. A. Lukas et al., Nucl. Phys. B552 (1999) 246, hep-th/9806051

Outline

Finally

- 11d supergravity on an interval gives Heterotic M-Theory
- Structure is fixed by supersymmetry and anomaly cancellation
- Hořava and Witten's formulation fails to treat $K_{\mu\nu}$ correctly and is led to divergences
- Moss's version treats $K_{\mu\nu}$ correctly and avoids divergences
- The consequences of the improved version for phenomenology are being investigated