The Refractive Index of Spacetime

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Motivation

- Drummond and Hathrell (1980ish).
- Low frequency effective action for QED in general spacetimes
- Showed superluminal velocity of light
- Two measures of speed of light used here, v_{ph} and v_{wf} .
- With standard disperson relations $\Rightarrow$ breakdown in causality

What's Gone Wrong

- Dispersion relations wrong?
- "Stable Causality"?
- Spacetime refractive index ($n = c/v_{ph}$) has gain?

Our Paper

- arXiv (hep-th/0905.0771)
- Work with T.Hollowood and G.Shore (both Swansea)
- Calculates refractive index for all frequencies (colours) of light
- Extends earlier work by TH and GS to spinor QED

Penrose Limits and the Eikonal Approximation

- Penrose Limits:
- "Spacetime as seen at the speed of light"
- Always gives a planewave (c.f. pp-waves)
- Eikonal or WKB approximation:
- $\omega \gg \sqrt{R}$
- $A_\mu(x) = \varepsilon_\mu(x) e^{-i\Theta(x)}$

Method

- General spacetime Maxwell equation
- Eikonal Mode $\Rightarrow$ off-shell
- Vacuum polarisation is equated to this
- $A^{\mu}(x)\Pi_{\mu\nu}(x,x')A^{\nu}(x')$

Method II

- Need to calculate vacuum polarisation
- Can not use momentum space techniques

$$\Pi \propto \int dx'^4 \sqrt{g(x')} G(x,x') G(x,x') \phi(x')$$

Method Part III

- $$n_{ij}(u;\omega) = \delta_{ij} - \frac{\alpha}{2\pi\omega^2}\int_0^{\infty - i\epsilon}\frac{dT}{T^2}\, i e^{-im^2 T} \times \int_0^1 d\xi \left[\delta_{ij} - \Delta_{ij}(u,u')\sqrt{\det \Delta_{ij}(u,u')}\right]_{u'=u-2\omega T\xi(1-\xi)}$$
- $\det \Delta_{\mu\nu}(x,x')$ is the VVM determinant, geometric.

Example Spacetimes I

- Penrose Limit Examples:
- dS/AdS, radial BH are flat
- ESU, $AdS_5 \times S^5$, are symmetric planes waves (Ricci/Conformally Flat)
- FRW, BH critical geodesics are Singular symmetric plane waves

Example Spacetimes II

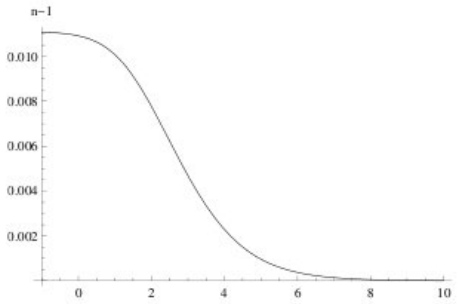
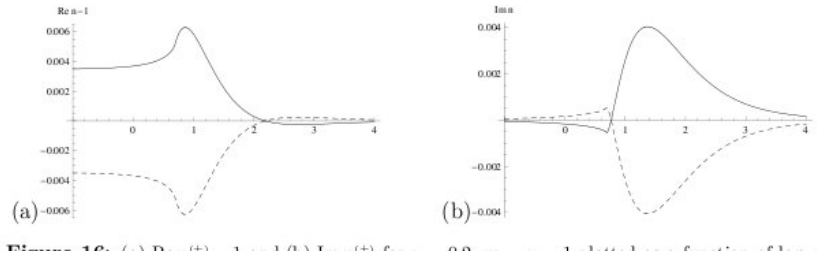


Figure 14: The refractive index $n(\omega) - 1$ for the conformally flat symmetric plane wave in units of $\alpha\sigma^2/(\pi m^2)$, as a function of $\log \omega\sigma/m^2$.

- The numerical results



Outlook

- Calculations in Spacetimes Before Penrose Limits
- Solving issue of photo multiplication
- Effects on massive particles or gravitons
- Extending pictue to AdS/CFT

The End

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- Thanks