Preserving Symmetries in Seiberg Dualities

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James Barnard

Department of Mathematics, Durham University

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Durham University

Vague motivation

- Seiberg duality[1] in $\mathcal{N} = 1$ SUSY gives us a different way of looking at supersymmetric gauge theories.
- We believe it will help in understanding many aspects of BSM physics such as gauge unification, proton decay and dynamical SUSY breaking.
- Problem: currently, dualities only exist for theories with highly constrained matter content and unrealistic superpotentials.
- Our goal is to find a dual theory to a more realistic GUT, like a supersymmetric $\mathrm{SU}(5)$ model.

[1] For a review: K. Intriligator, N. Seiberg - arXiv:hep-th/9509066

Seiberg duality in a nutshell

Original theory - SQCD with N colours and F_Q flavours

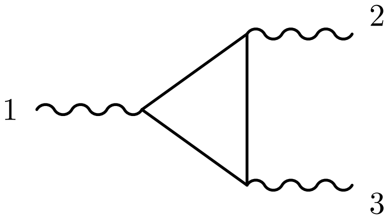
	$\mathrm{SU}(N)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$
Q	$\mathbf{N}$	$\mathbf{F_Q}$	$\mathbf{1}$
$\tilde{Q}$	$\overline{\mathbf{N}}$	$\mathbf{1}$	$\mathbf{F_Q}$

Dual theory - SQCD+M with $n = F_Q - N$ colours and F_Q flavours

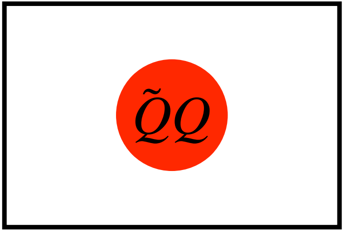
	$\mathrm{SU}(n)$	$\mathrm{SU}(F_Q)_L$	$\mathrm{SU}(F_Q)_R$
q	$\mathbf{n}$	$\overline{\mathbf{F_Q}}$	$\mathbf{1}$
$\tilde{q}$	$\overline{\mathbf{n}}$	$\mathbf{1}$	$\overline{\mathbf{F_Q}}$
M	$\mathbf{1}$	$\mathbf{F_Q}$	$\mathbf{F_Q}$

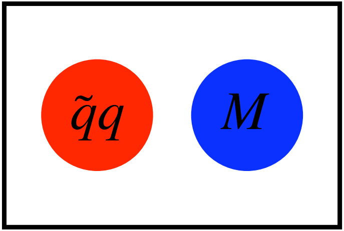
Tests of the duality

- The global symmetries of both theories are the same.
- The classical moduli spaces of both theories are the same (i.e. the mesons and baryons match).
- The duality is preserved under deformations, e.g. quark mass terms.
- Highly non-trivial 't Hooft anomaly matching conditions are satisfied for non-anomalous global symmetries, especially those involving the R-symmetry.



Mesons and superpotential





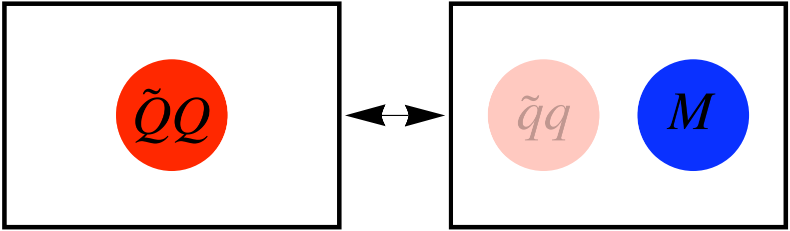
Original theory - $\mathrm{SU}(N)$

- ${F_Q}^2$ mesons $\tilde{Q}Q$

Dual theory - $\mathrm{SU}(F_Q - N)$

- ${F_Q}^2$ composite mesons $\tilde{q}q$
- ${F_Q}^2$ elementary mesons M

Mesons and superpotential



Original theory - $\mathrm{SU}(N)$

- ${F_Q}^2$ mesons $\tilde{Q}Q$

Dual theory - $\mathrm{SU}(F_Q - N)$

- $W_{\mathrm{dual}} = M\tilde{q}q$
- F-terms give $\tilde{q}q = 0$
- ${F_Q}^2$ elementary mesons M

Adding more matter: the KSS model

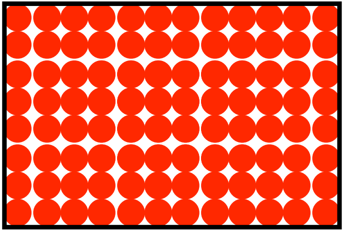
- To find a dual GUT, we need to be able to find dualities for theories with adjoint and/or antisymmetric representations of the gauge group.
- Consider adding an adjoint X to the original theory[2]. The mesons are now

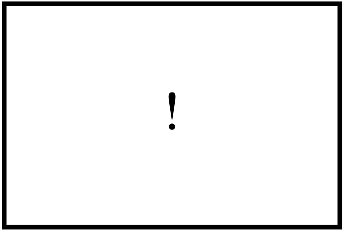
$$M_j = \tilde{Q} X^j Q$$

for **any** positive integer j.

[2] D. Kutasov, A. Schwimmer, N. Seiberg - arXiv:hep-th/9510222

The KSS model





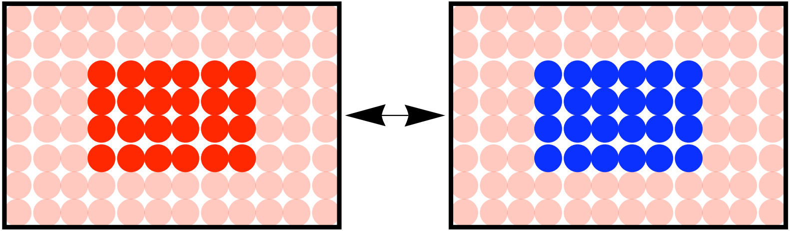
Original theory - $\mathrm{SU}(N)$

- ∞ mesons $\tilde{Q}X^jQ$

Dual theory - $\mathrm{SU}(!)$

- Argh!

The KSS model



Original theory - $\mathrm{SU}(N)$

- $W_{\mathrm{orig}} = X^{k+1}$
- F-terms give $X^k = 0$
- $kF_Q{}^2$ mesons $M_j = \tilde{Q}X^jQ$
- $j = 0, \ldots, k-1$

Dual theory - $\mathrm{SU}(kF_Q - N)$

- $W_{\mathrm{dual}} = x^{k+1} + \sum_j M_j \tilde{q} x^{k-1-j} q$
- F-terms give $x^k = \tilde{q}x^jq = 0$
- $kF_Q{}^2$ elementary mesons M_j
- $j = 0, \ldots, k-1$

Preserving symmetries

- Adding a superpotential to the original theory is often necessary, but reduces the number of global symmetries.
- If there are too few global symmetries we cannot test the duality properly.
- In particular, it seems very important for the theory to retain an R-symmetry to ensure non-trivial 't Hooft anomaly matching conditions.
- By adding gauge singlets to the superpotential we can generally retain an R-symmetry.

Example: the deformed KSS model

- Consider the model just discussed: SQCD with N colours, F_Q flavours and an extra adjoint field X.
- This model required superpotential $W_{\mathrm{orig}} = X^{k+1}$ for a duality to make sense.
- Now deform the superpotential with an extra term

$$W_{\mathrm{orig}} = s_0 X^{k+1} + s_l X^{k+1-l} \quad \text{with} \quad 1 \leq l \leq k-1$$

  (this theory now exhibits GUT-like behaviour).
- R-symmetries require $R(W) = 2$. Hence the deformed KSS model has no R-symmetry.
- The dual theory still exists and is of the same form, but with a more complicated superpotential.

The R-symmetric deformed KSS model

- A non-anomalous R-symmetry can be restored by adding a gauge singlet ϕ and setting

$$W_{\rm orig} = \phi^{\rho_0} X^{k+1} + \phi^{\rho_l} X^{k+1-l}$$

- This gives

$$R(X) = \frac{2\left(\rho_l - \rho_0\right)}{\rho_l\left(k+1\right) - \rho_0\left(k+1-l\right)}, \qquad R(\phi) = \frac{l}{\rho_l - \rho_0} R(X)$$

  and therefore $R(W) = 2$.

- One can think of this process as elevating all coupling constants in the superpotential to fields $s \longrightarrow \phi^{\rho}$.

A meson ambiguity

- Having introduced a new, gauge invariant field the most general meson is now

$$M_j^{(\alpha)} = \tilde{Q}(\phi^\alpha X)^j Q \quad \text{with} \quad j = 0, \ldots, k-1$$

  for arbitrary α.
- Different values of α give different R-charges to the mesons, but these R-charges appear in the 't Hooft anomaly matching conditions so it's important to get them right!

Fixing the meson ambiguity

- In the undeformed dual theory, the elementary mesons project out the composite mesons with the terms

$$W_{\text{dual}} \supset \sum_j M_j \tilde{q} x^{k-1-j} q$$

- We propose that the singlets are shared evenly among the original and dual mesons, i.e. we attach the same powers of ϕ to adjoints on both sides of the duality.
- The meson terms in the deformed dual theory now look like

$$\sum_j M_j^{(\alpha)} \left[\tilde{q}(\phi^\alpha x)^{k-1-j} q\right] = \sum_j \left[\tilde{Q}(\phi^\alpha X)^j Q\right] \left[\tilde{q}(\phi^\alpha x)^{k-1-j} q\right]$$

- Solving $R(W_{\text{dual}}) = 2$ for α unambiguously fixes its value and therefore the definition of the mesons.

Comments

- Attaching ϕ^α to all adjoints also results in the exact powers required to match baryons between the two theories.
- This is an independent test and follows from known results by considering ϕ as an elevated coupling constant.
- Demanding that the $\mathrm{U}(1)_R{}^3$ 't Hooft anomalies match restricts the choices for the powers of ϕ appearing in the superpotential.

ρ_0	0	any	any
ρ_l	any	$\left(1-\frac{l}{k-1}\right)\rho_0$	ρ_0

Other applications

- This technique can also be used to find new dualities with more general matter content and realistic superpotentials.[3]
- First, one writes down the superpotential required in the original theory to ensure that the number of mesons is well defined. This often breaks any R-symmetries.
- An R-symmetry is then restored using this technique. Hence a duality can be found and rigourously tested.
- We have demonstrated the idea for SQCD with an arbitrary number of adjoints and SQCD with three generations of antisymmetric tensor - an important step towards a dual of an $\mathrm{SU}(5)$ GUT.

[3] S. Abel, JB - arXiv:0903.1313[hep-th]

Summary

- Seiberg duality may have many phenomenological applications, but to exploit them we need dualities involving realistic theories.
- Adding more complicated matter representations to dualities often requires a non-zero superpotential in the original theory.
- If this superpotential breaks too many global symmetries (particularly R-symmetries) a duality cannot be rigourously tested.
- Adding gauge singlets (or equivalently elevating coupling constants to fields) can address this problem.
- However, one must be careful to include the gauge singlets properly when defining the mesons of the theory.
- This technique can be further expanded to allow us to find and test new dualities with semi-realistic matter content.