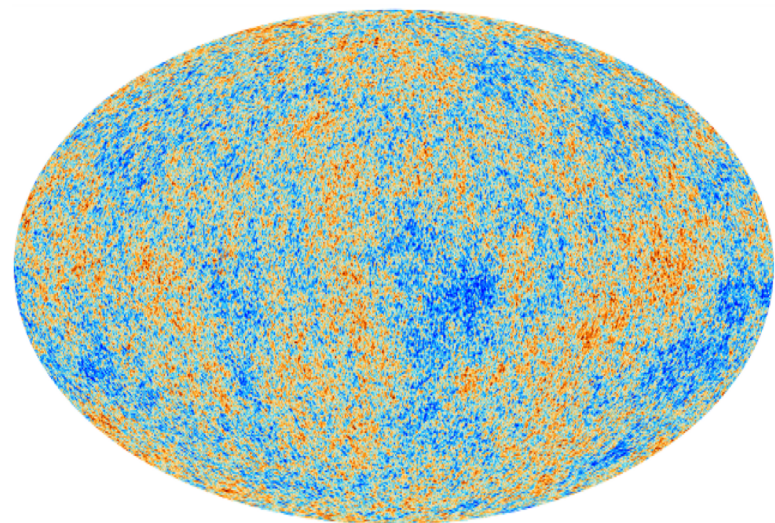
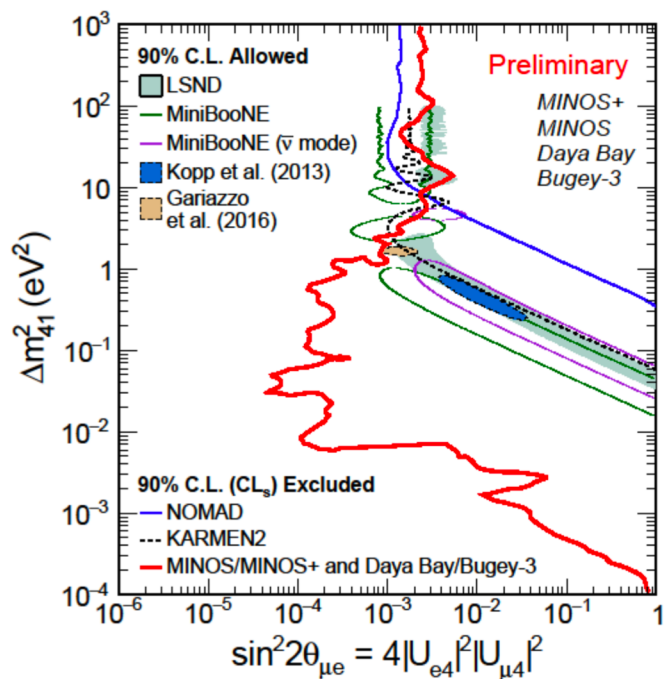
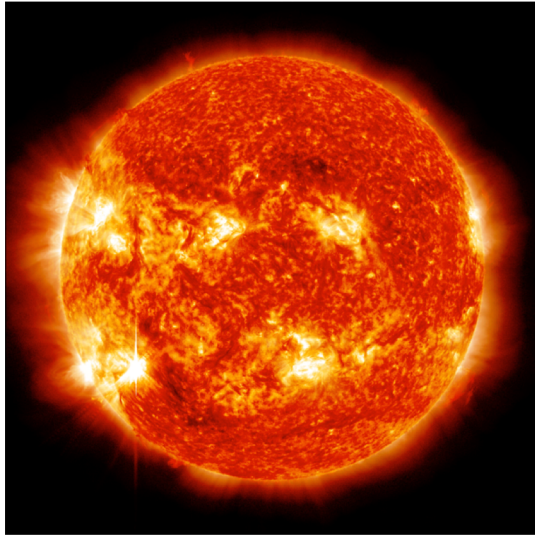


# A combined view of sterile-neutrino constraints from CMB and neutrino-oscillation measurements



Justin Evans  
University of Manchester

# Neutrino oscillation

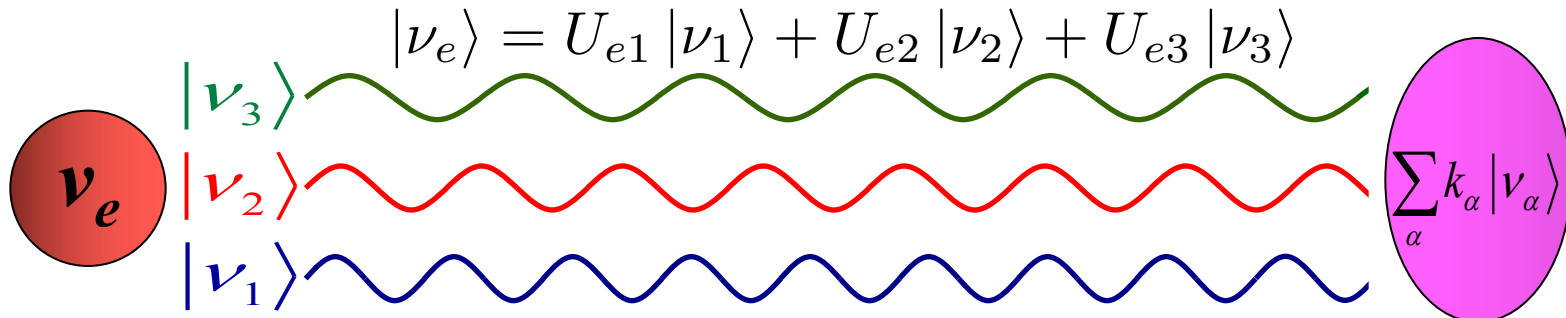


Neutrino created by a weak process:  
Eigenstate of flavour

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



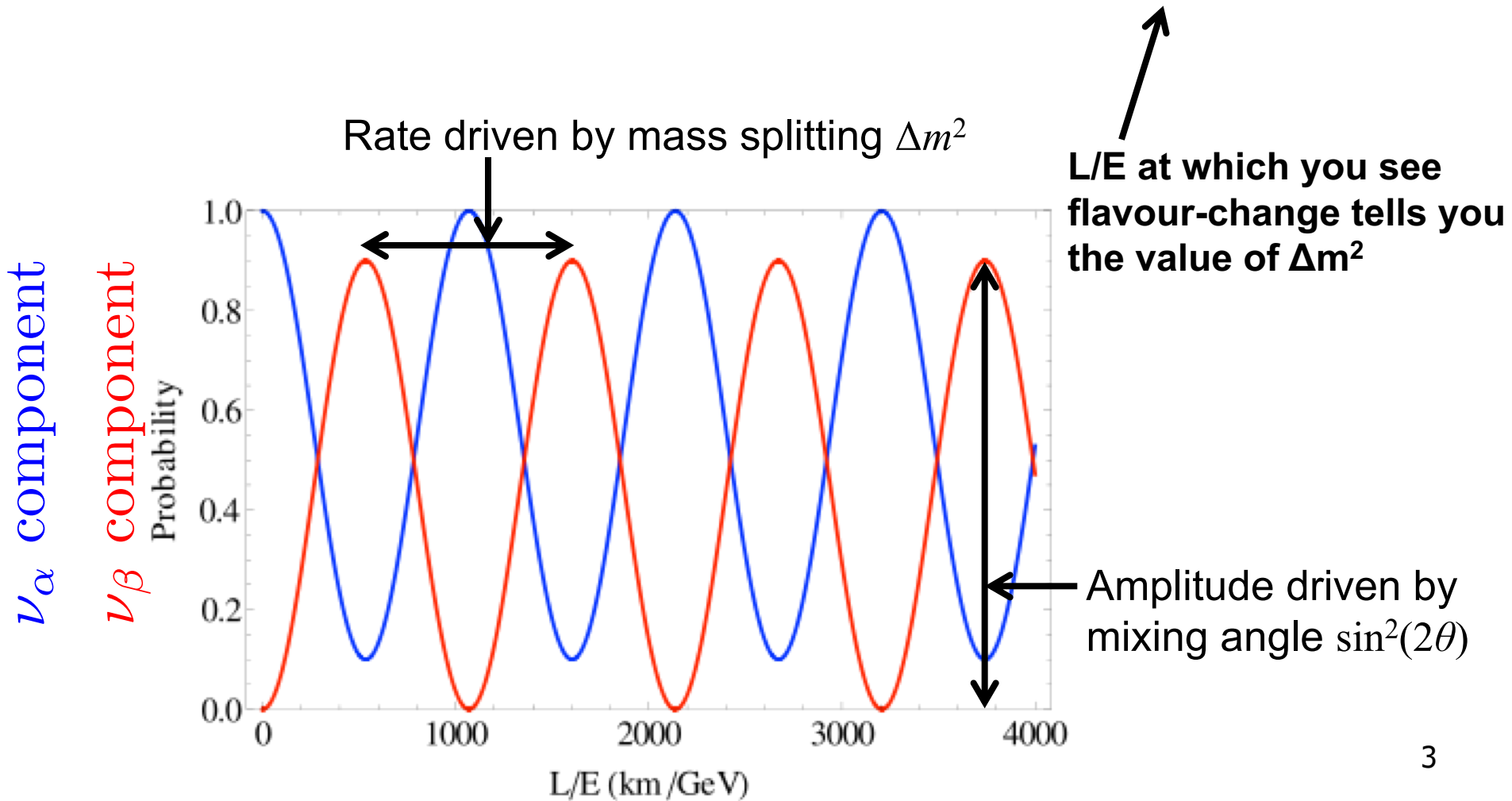
Does not arrive in a flavour eigenstate;  
Detection collapses wavefunction back into a flavour eigenstate



The relative phases of the mass eigenstates change

# Neutrino oscillations

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2(2\theta) \sin^2 \left( 1.27 \frac{\Delta m_{21}^2 [\text{eV}^2] L [\text{km}]}{E [\text{GeV}]} \right)$$



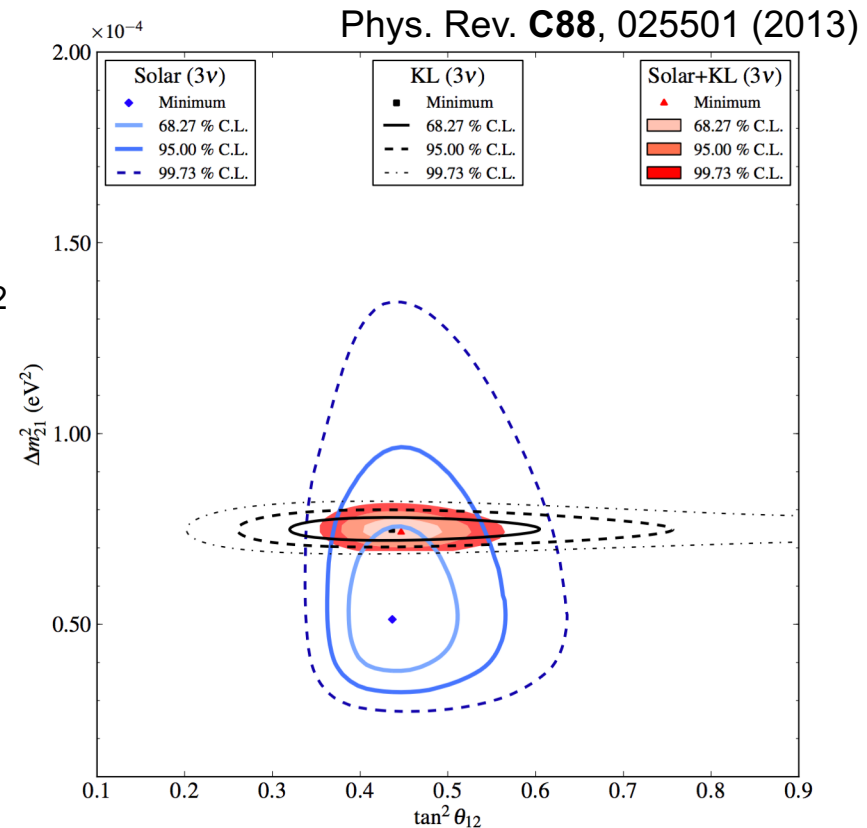
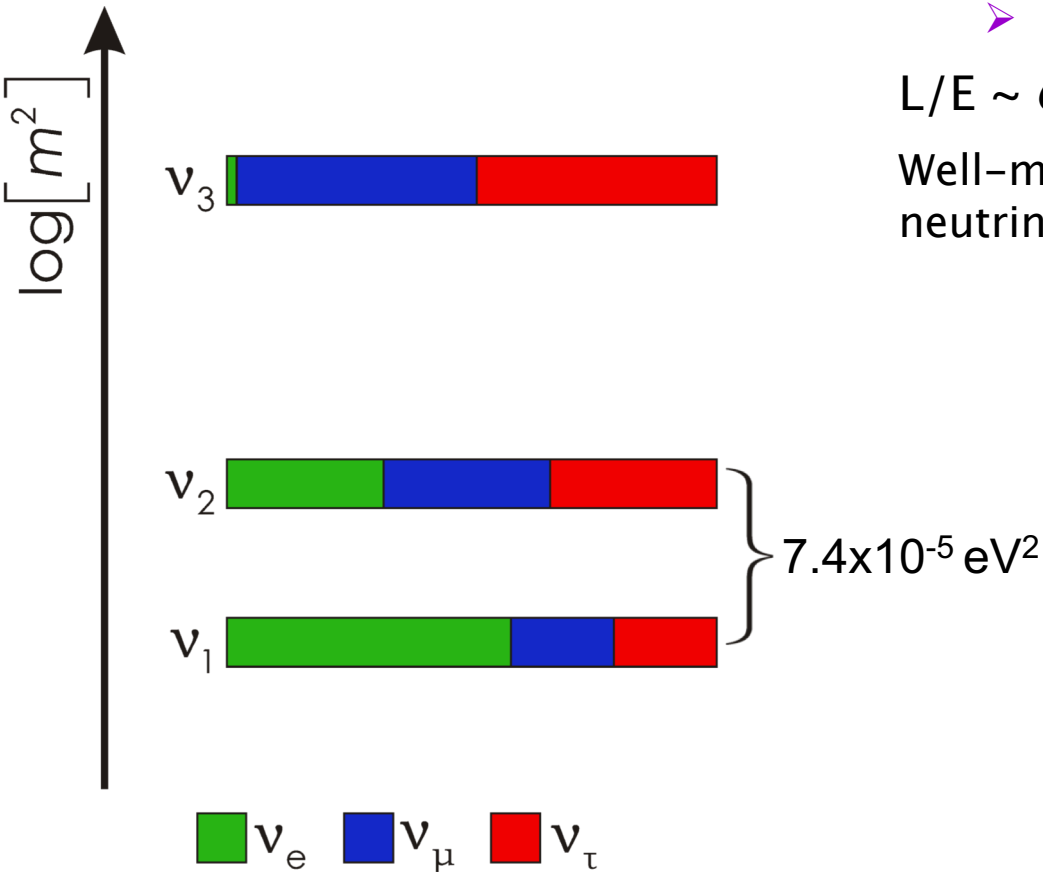
# The three-neutrino picture

Smallest mass splitting

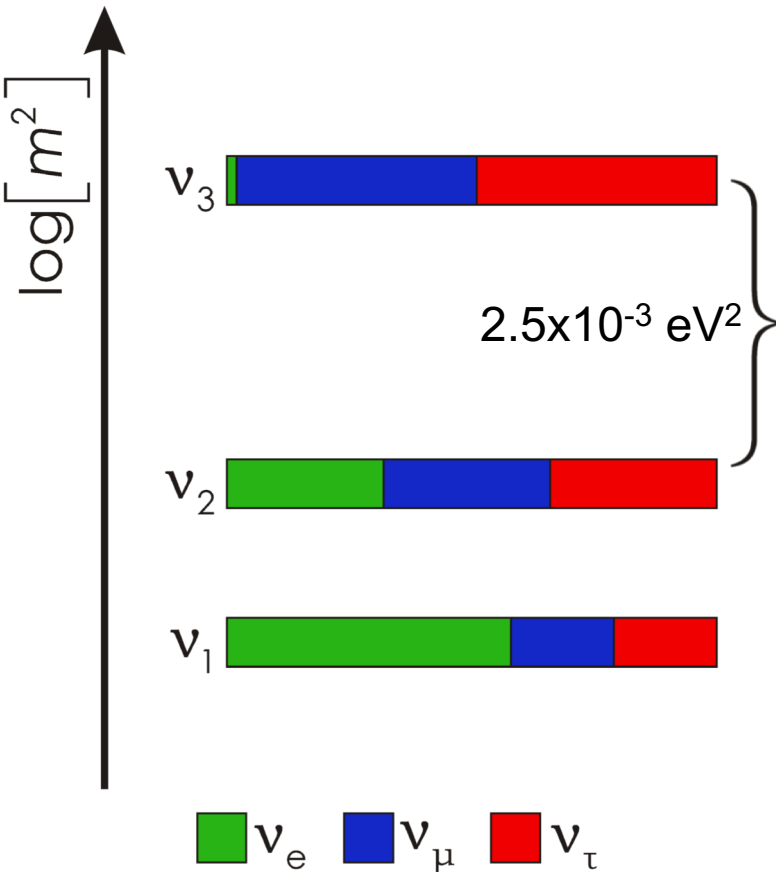
➤ ‘Solar’ mass splitting

$L/E \sim O(10^5 \text{ km/GeV})$

Well-measured with both solar and reactor neutrinos



# The three-neutrino picture

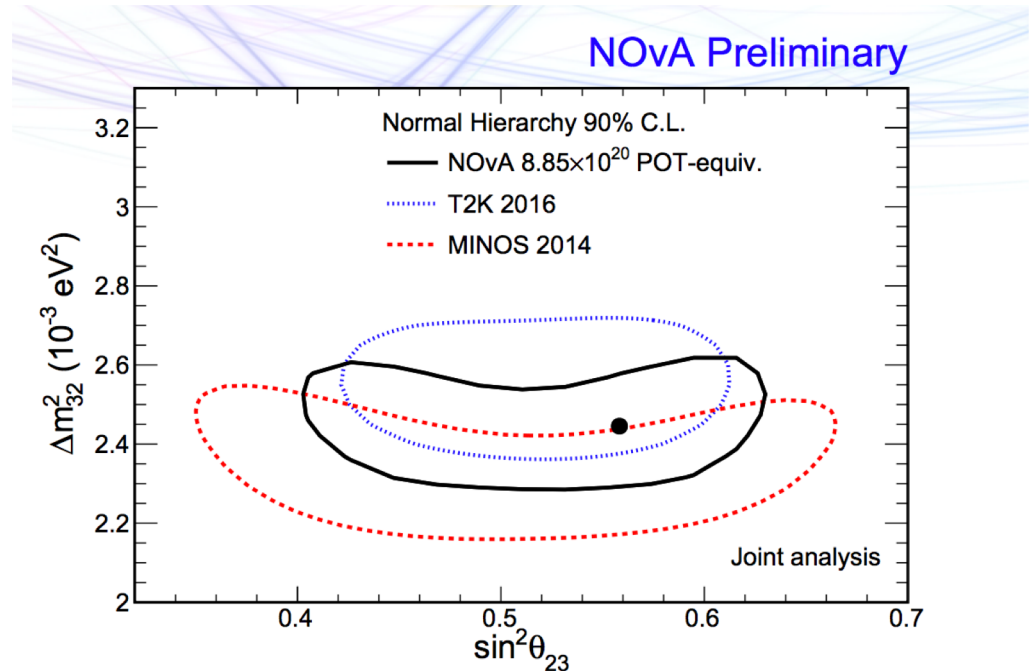


Largest mass splitting

➤ ‘Atmospheric’ mass splitting

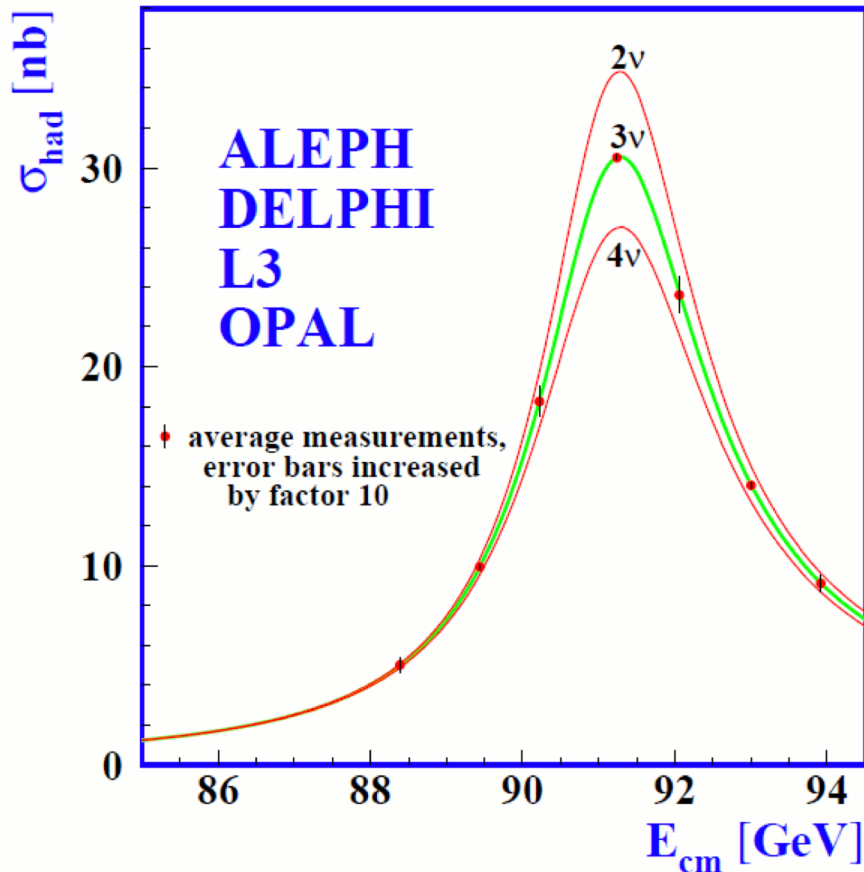
$L/E \sim O(10^3 \text{ km/GeV})$

Well-measured with atmospheric and accelerator neutrinos



# The three-flavour picture

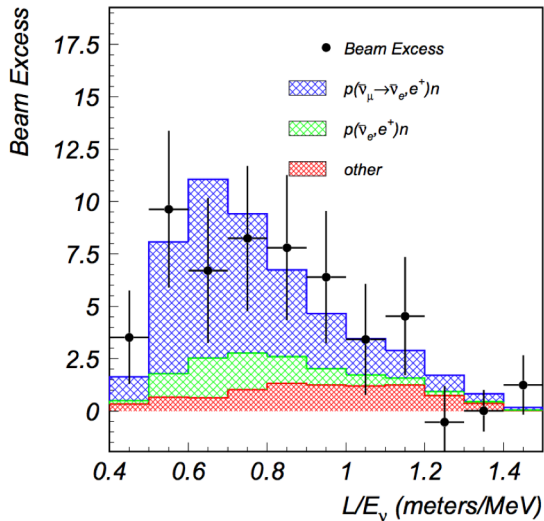
$$\Gamma_Z = \Gamma(Z^0 \rightarrow q\bar{q}) + 3\Gamma(Z^0 \rightarrow e^+e^-) + N_\nu\Gamma(Z^0 \rightarrow \nu\bar{\nu})$$



$e^+e^-$  scattering confirms  
our suspicions

➤  $N_\nu = 2.984 \pm 0.008$

# So what's the problem?

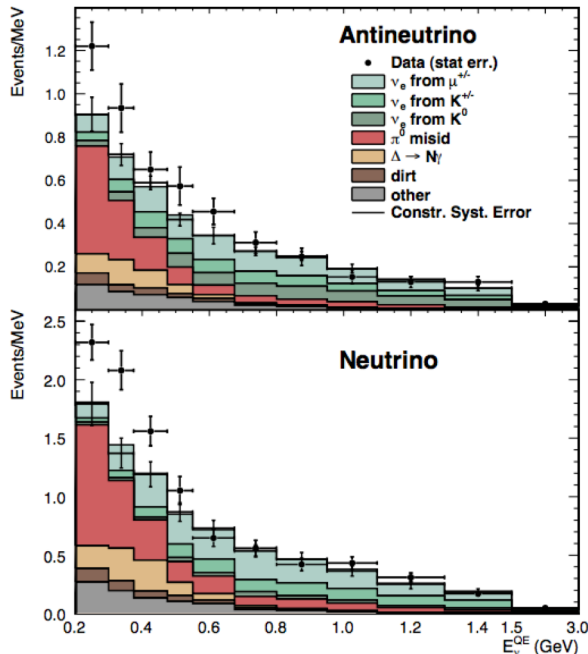


## LSND

An antielectron-like excess in a  $\bar{\nu}_\mu$  beam

$L/E = O(1 \text{ km} / \text{GeV})$

Phys. Rev. **D64**, 112007 (2001)



## MiniBooNE

Similar electron- and antielectron-like excesses in  $\nu_\mu$  and  $\bar{\nu}_\mu$  beams

$L/E = O(1 \text{ km} / \text{GeV})$

Phys. Rev. Lett. **110**, 161801 (2013)

This would require  $\Delta m^2 = O(1 \text{ eV}^2)$

➤ Does not fit with the three-flavour picture

# The light, sterile neutrino

$e^+e^-$  scattering tells us this neutrino doesn't couple with the Z boson

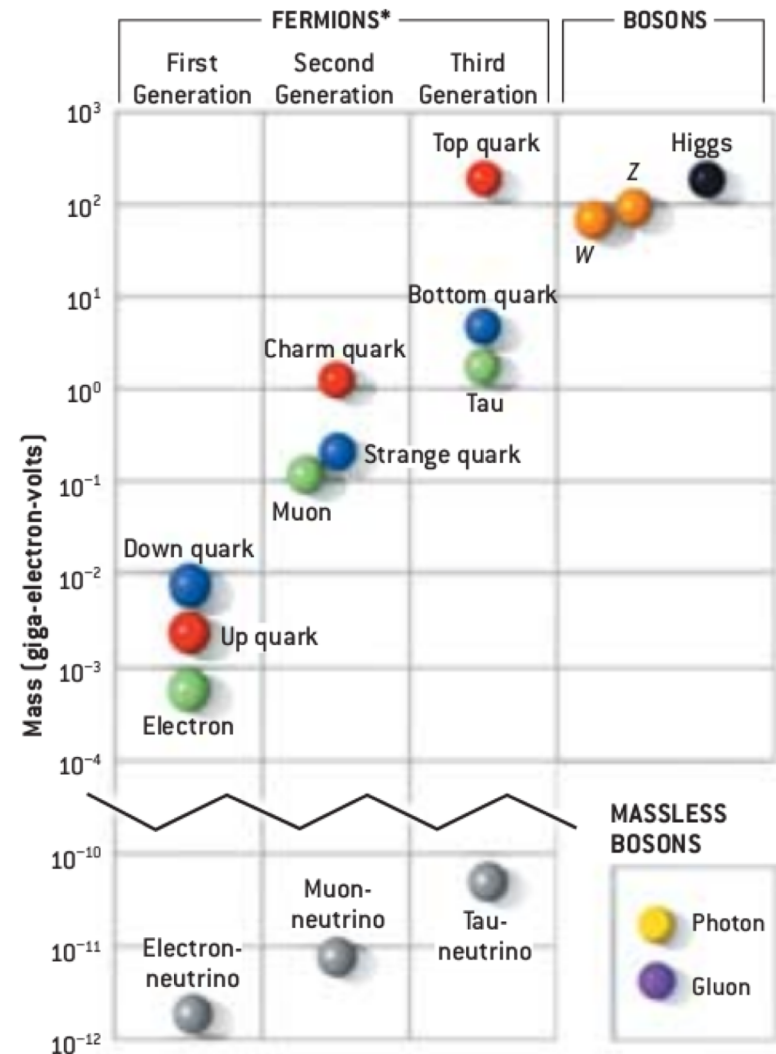
- Sterile – oscillations are the only way we can see it

'Light' means eV rather than keV (or above) scale

- Theorists love heavy sterile neutrinos, e.g. seesaw models with Majorana neutrinos

Sterile neutrinos aren't a crazy idea

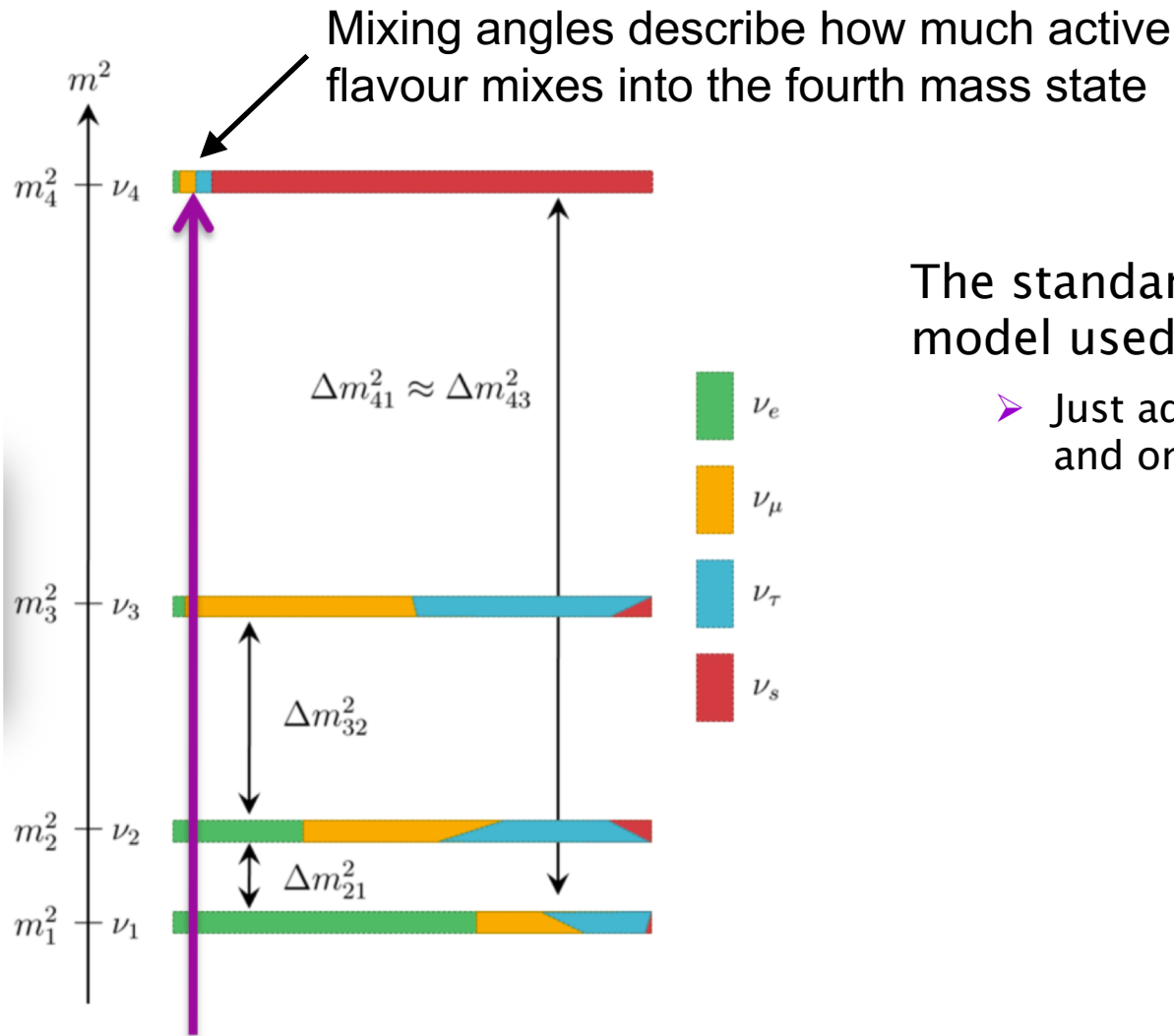
- Neutrinos are light, are the only neutral fermion, and only spin one way
- There's definitely something odd about them, suggesting new physics is involved somewhere





# The 3+1 model

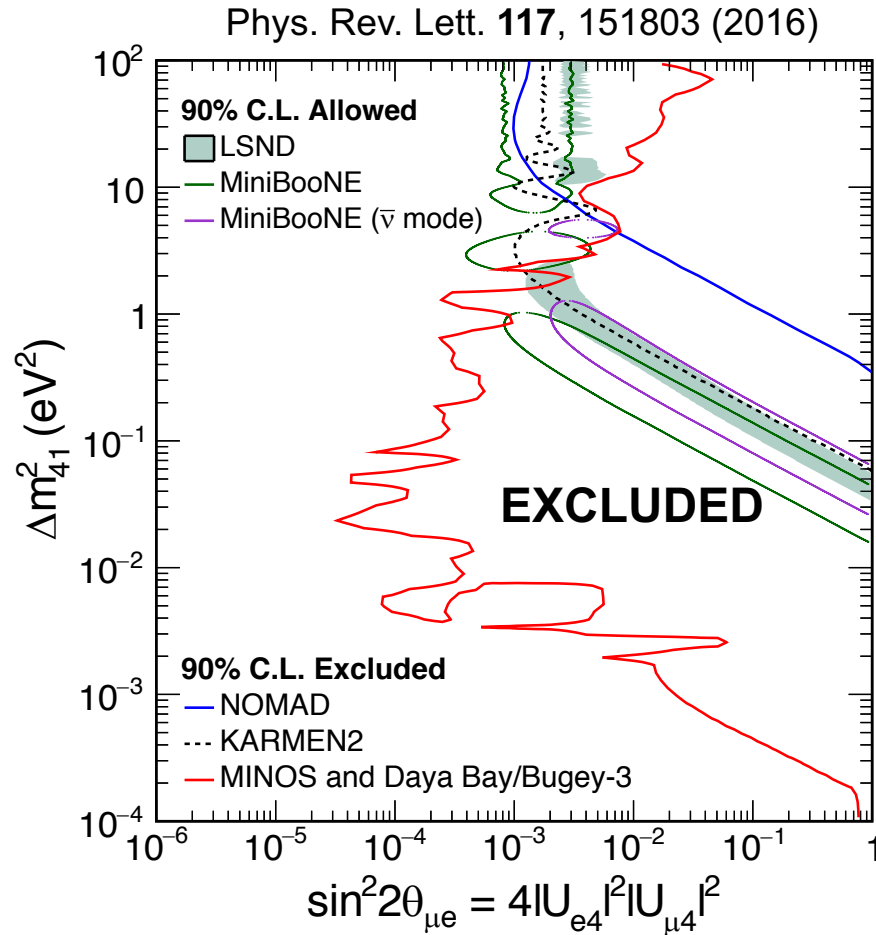
$\theta_{14}$ : electron flavour  
 $\theta_{24}$ : muon flavour  
 $\theta_{34}$ : tau flavour



The standard phenomenological model used to compare data sets

- Just add one new mass eigenstate and one sterile flavour state

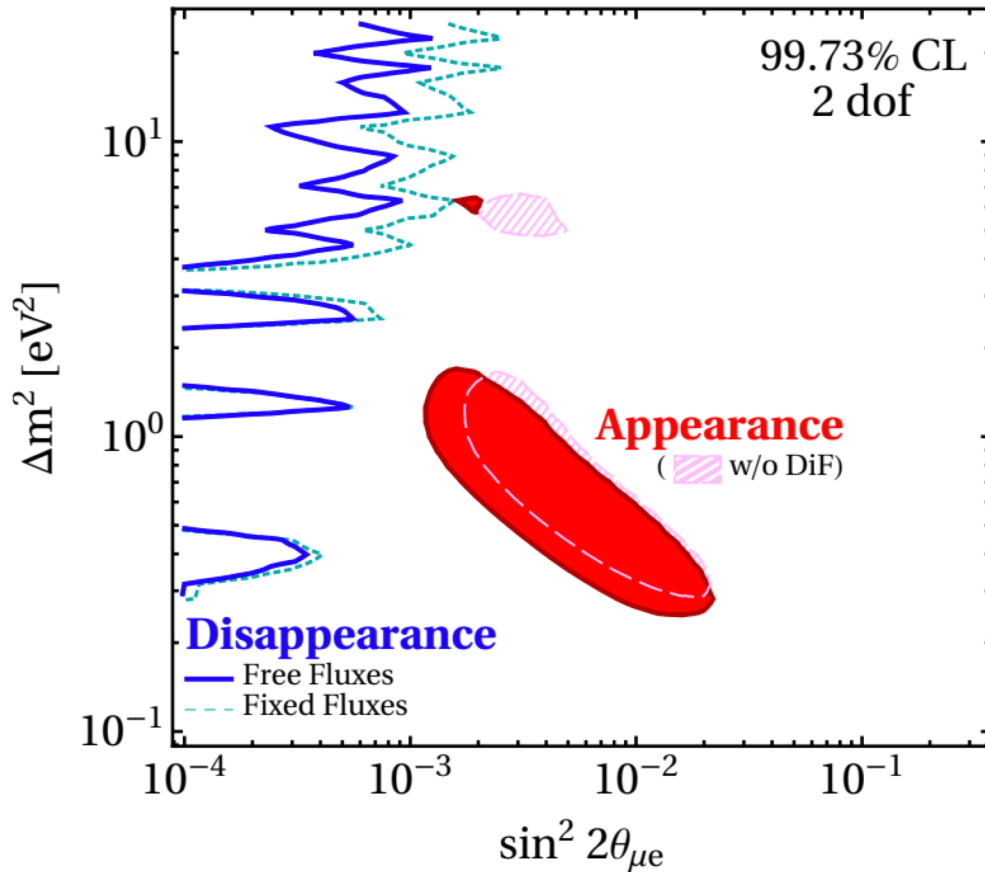
# Contention



- LSND and MiniBooNE see sterile neutrinos
- Some gallium and reactor measurements also favour a sterile neutrino
- Other experiments rule out most of their parameter space

# The 3+1 model

M. Dentler *et al.*, arXiv:1803.10661



The standard phenomenological model used to compare data sets

- Just add one new mass eigenstate and one sterile flavour state

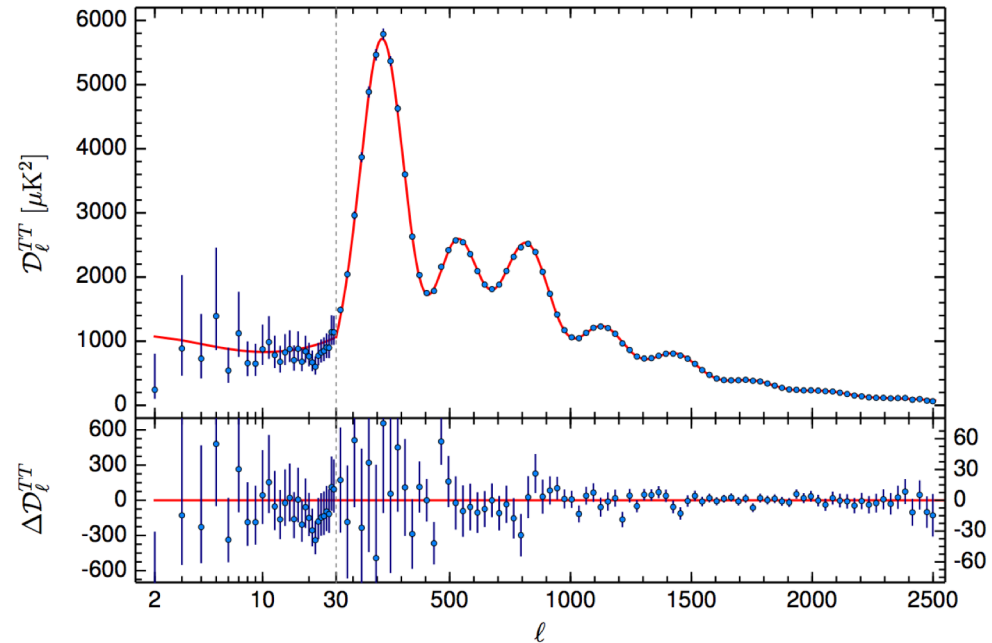
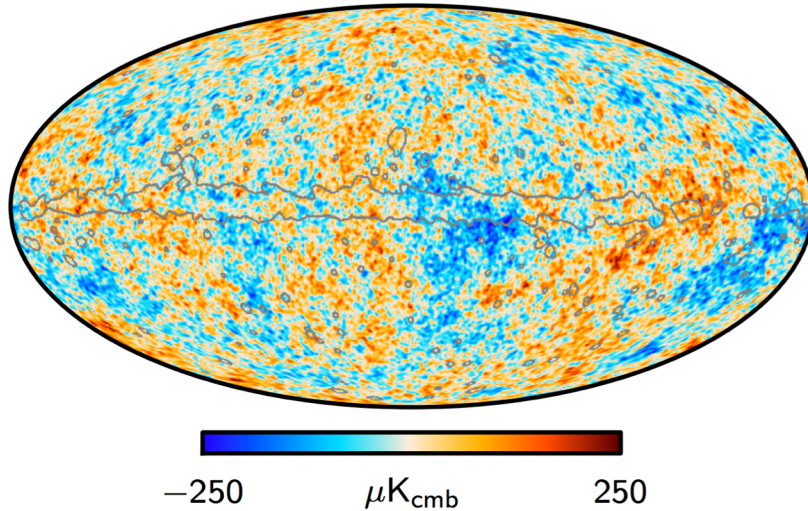
Not possible to explain all data using this model

Can of course move to 3+2, 3+3 models...

- Still hard to reconcile all data
- But you can always keep adding parameters

# Cosmic Microwave Background

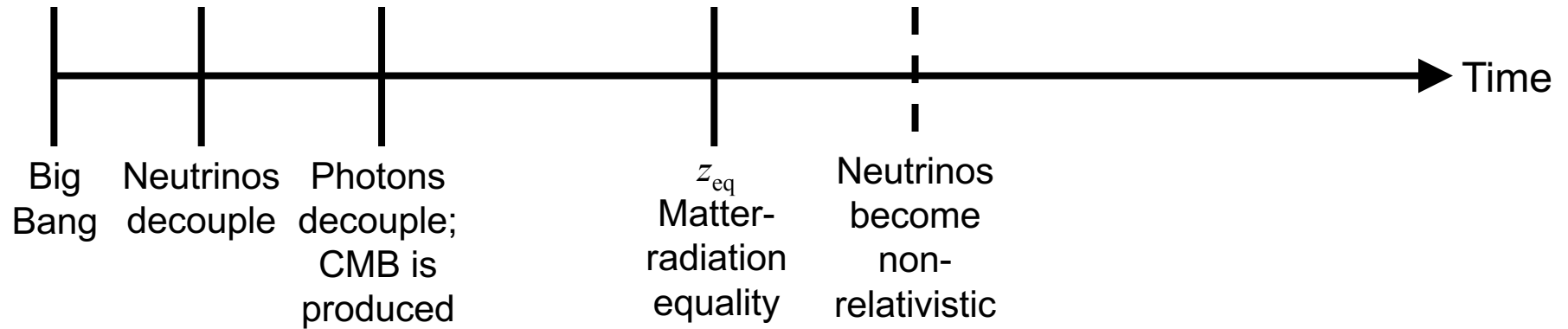
Planck: A&A **594**, A11 (2016)



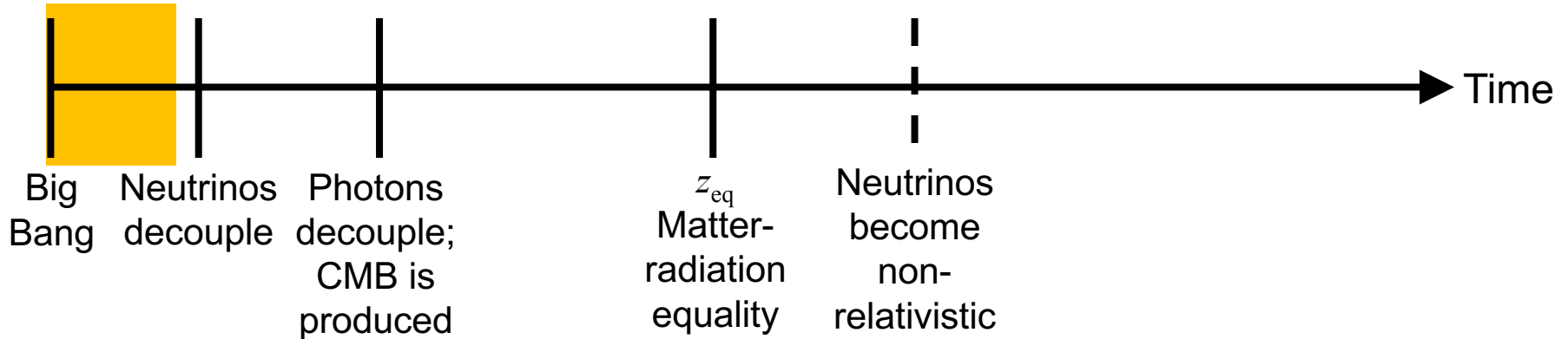
Cosmic Microwave Background provides another constraint on the existence of sterile neutrinos

- Specifically the power spectrum – angular size of fluctuations
- Not usually compared directly to the particle physics constraints

# Cosmology



# Neutrino sea



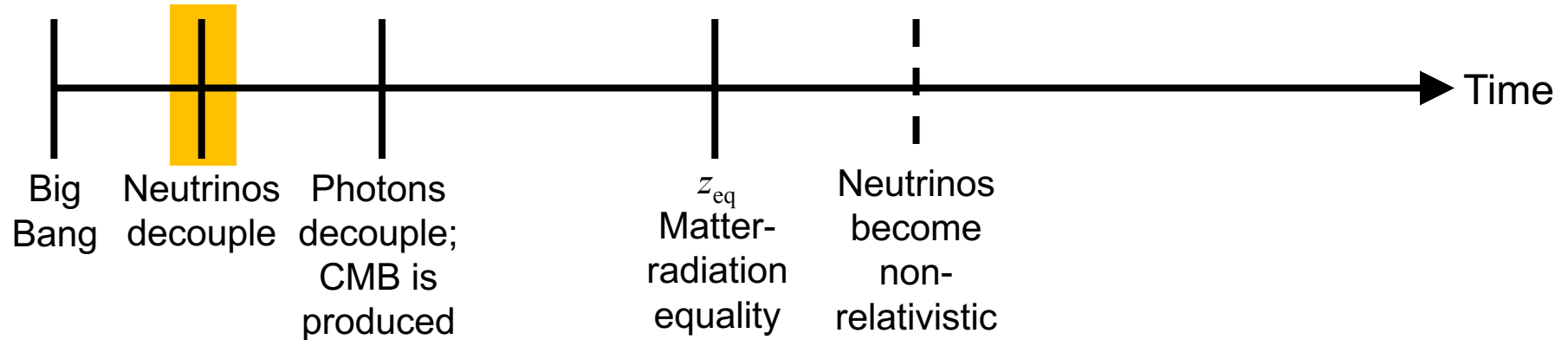
## Frequent weak interactions in the early universe

- $\nu_{e,\mu,\tau}$  kept in thermal equilibrium
- Momentum spectrum has a Fermi-Dirac form:

$$f_{\text{eq}}(p, T) = \frac{1}{e^{\frac{(p-\mu_\nu)}{T}} + 1}$$

- ( $\mu_\nu$  is a chemical potential, only exists if there is a neutrino-antineutrino asymmetry)
- With this function and some statistical mechanics, various properties of the neutrino sea can be calculated

# Neutrino decoupling



$T \sim 1 \text{ MeV}$

- Neutrinos decouple, but are still relativistic (i.e. radiation)
- Slightly flavour-dependent since there are more electrons around than muons or taus
- Fermi-Dirac distribution is frozen in, and then redshifts

$N_{\text{eff}}$  is the number of relativistic degrees of freedom in this radiation sea

A short time later  $T < m_e$  and  $e^+e^-$  pairs annihilate to photons

- This increases the temperature of the CMB
- The neutrino-electron interactions produce percent-level fluctuations to the high-energy part of the neutrino momentum distribution
- Increases  $N_{\text{eff}}$  from 3 to 3.046

# The expanding universe

## Friedman equation

- $a$  is the size of the universe
  - $\rho$  is the energy density
  - $p$  is the radiation pressure
- $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3p}{c^2} \right)$$

$$\rho = \underbrace{\rho_\gamma + \rho_\nu}_{\rho_r} + \rho_{\text{CDM}} + \rho_b + \rho_\Lambda$$

$\rho_r$ : radiation energy density (before neutrinos become non-relativistic)

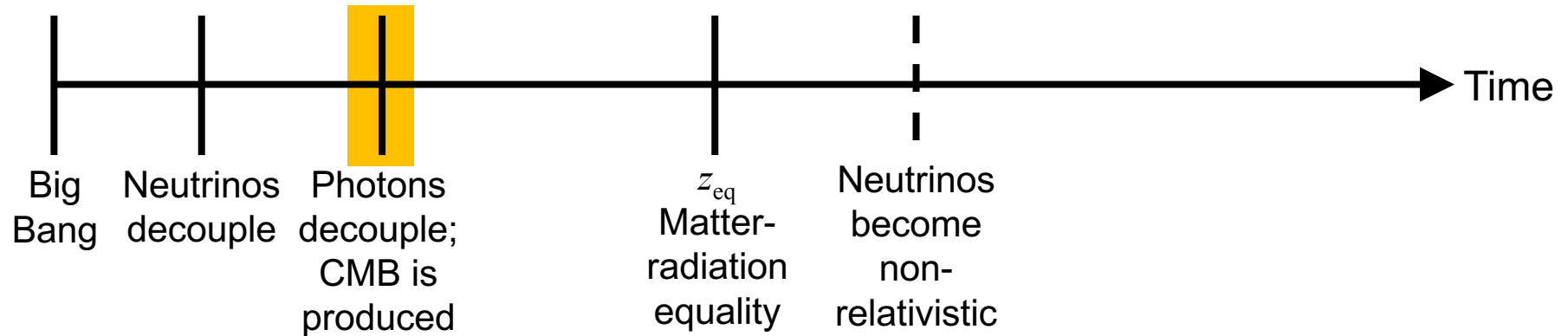
$$\rho_r = \rho_\gamma + \rho_\nu = \left[ 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{\frac{4}{3}} N_{\text{eff}} \right] \rho_\gamma$$

Can relate the neutrino energy density to the photon energy density

- Then measure the photon properties from the CMB
- And your remaining free parameter is  $N_{\text{eff}}$



# Birth of the Cosmic Microwave Background



Photons decouple and the CMB is produced

- Fluctuations are frozen in

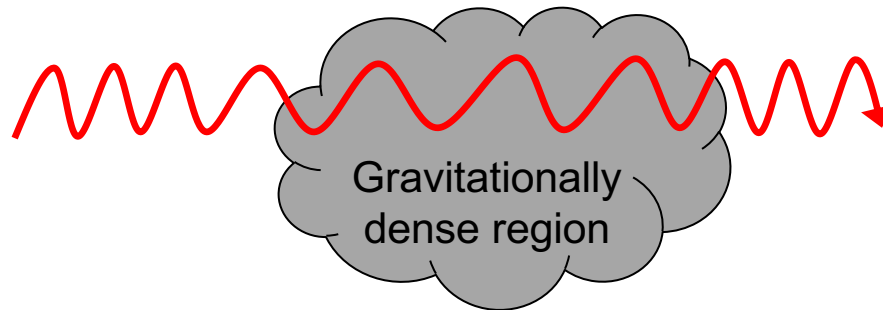
An increase in  $N_{\text{eff}}$ , or neutrino mass, can change when this happens

- Changes the size of the fluctuations on the sky

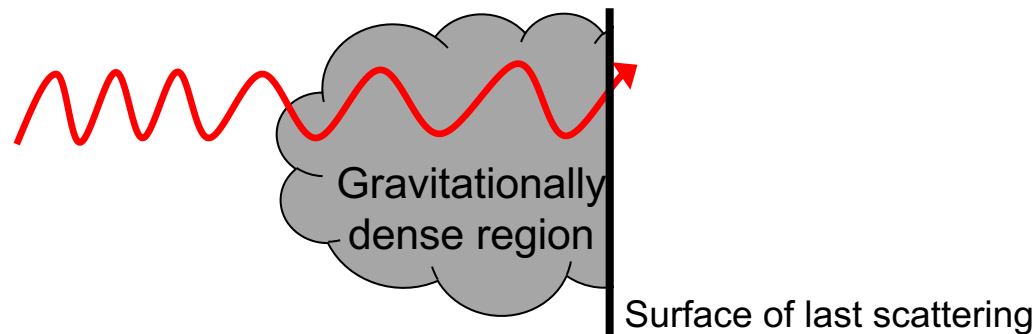
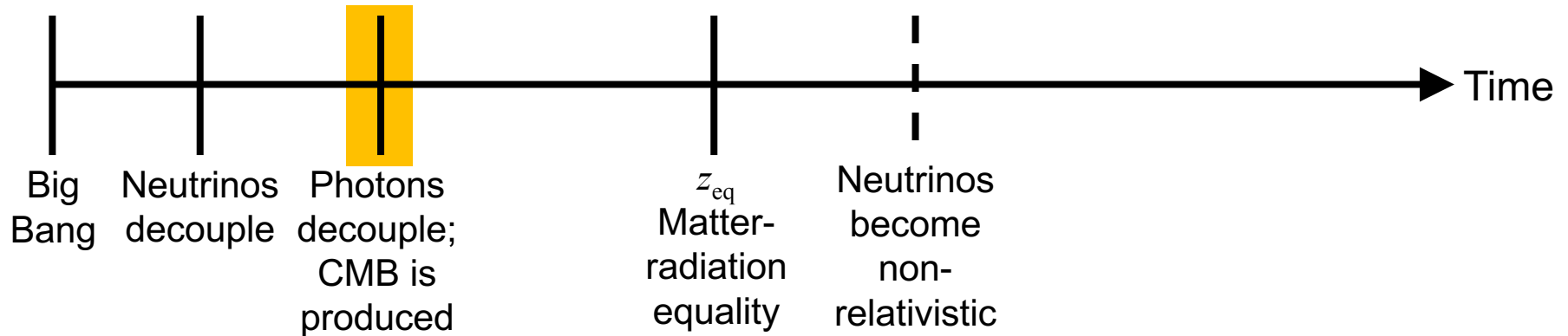
# Sachs–Wolfe effect

Photon is red-shifted on the way in, blue shifted on the way out

- The end result is uninteresting: a photon with the same wavelength as it started
- But there are situations where the end result is more interesting...



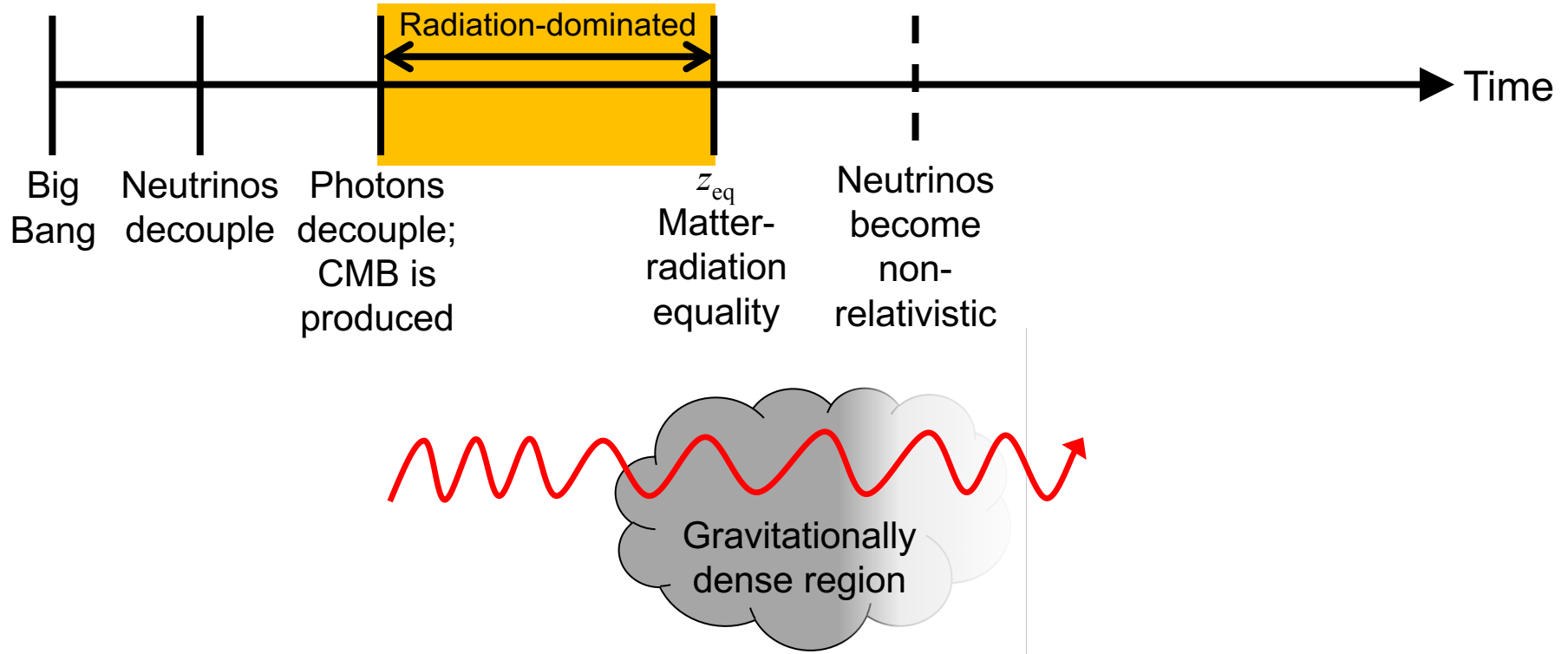
# Sachs-Wolfe effect



Gravitational fluctuations at moment the CMB is produced

- Red / blue shift is frozen in

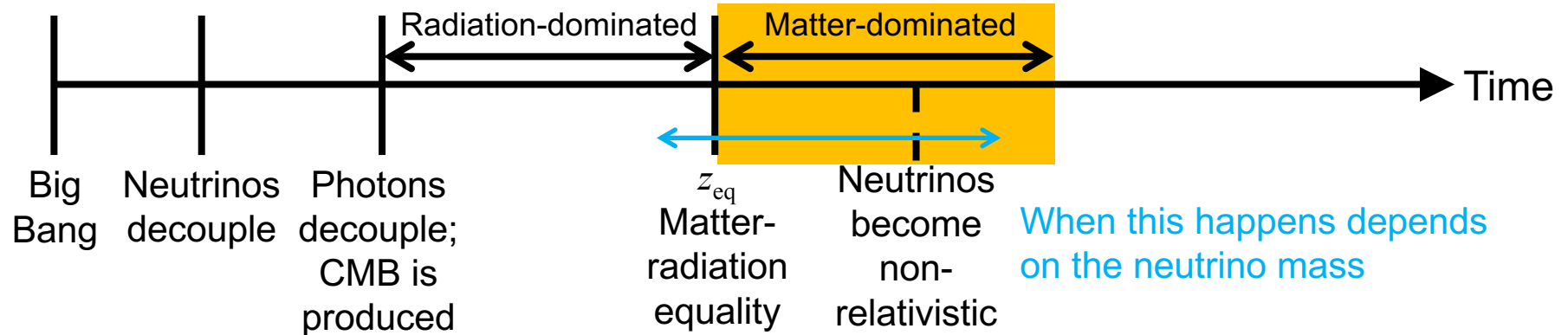
# Early-time integrated Sachs–Wolfe effect



Radiation modifies the regional gravitational perturbation as the photon passes through

- The photon gains an overall redshift or blueshift
- Neutrinos with masses below 1 eV are radiation at this point

# Matter-dominated universe



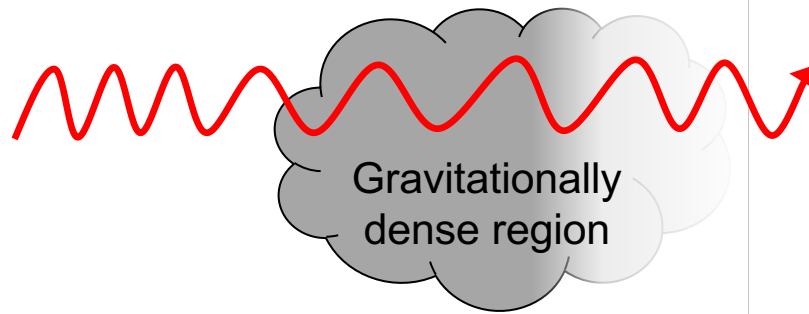
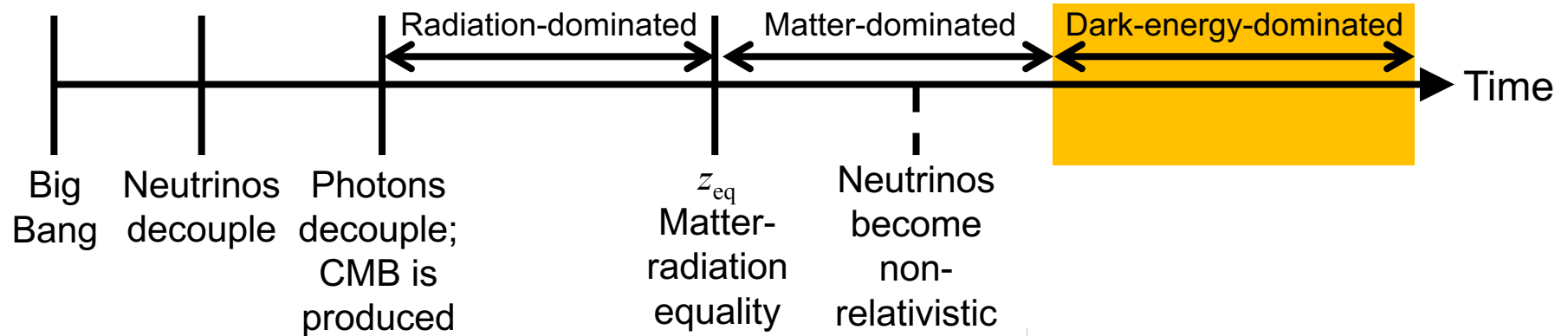
Gravitational perturbations are fairly constant

- No integrated Sachs–Wolfe effect

The presence of more or less non-relativistic matter (neutrinos) affects how much the universe has expanded since the CMB was produced

- Changes the angular size of the fluctuations
- Depends on the neutrino mass

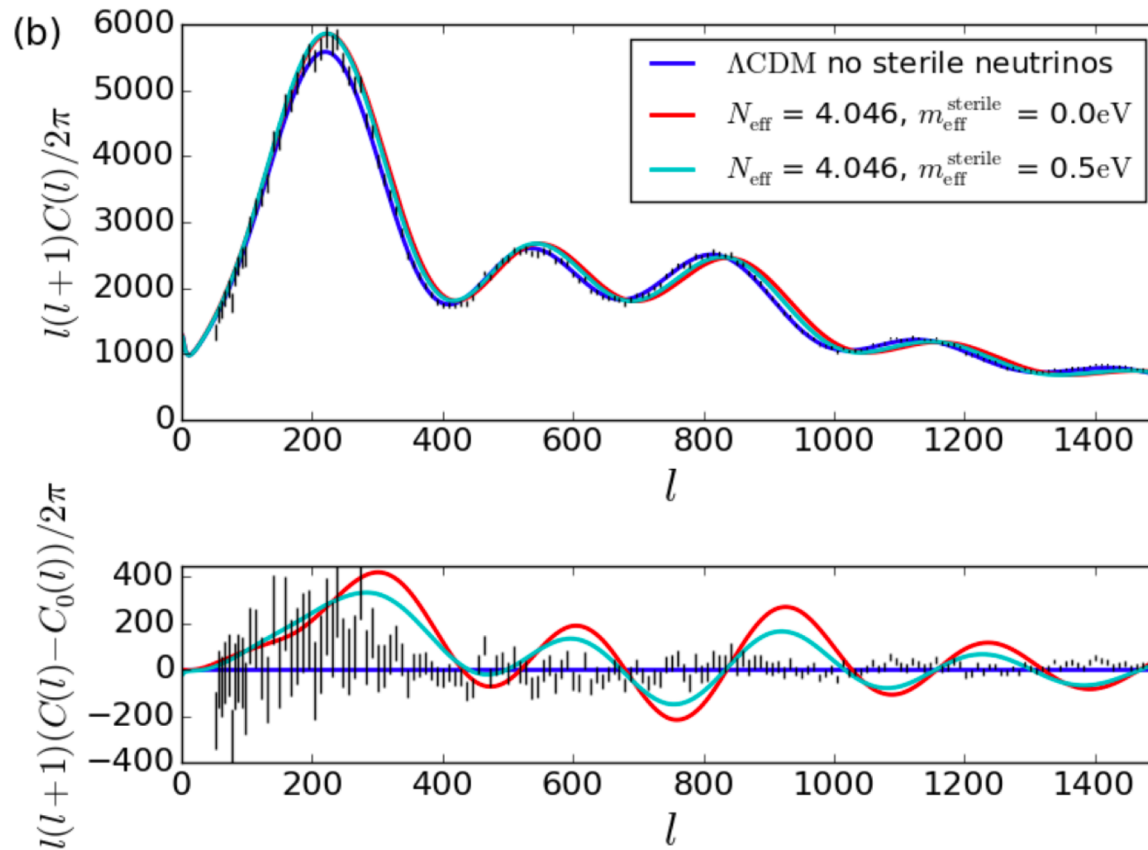
# Late-time integrated Sachs–Wolfe effect



Dark energy washes out the gravitationally dense regions (superclusters) as photons pass through

- A redshift or blueshift can be frozen in
- The presence of additional non-relativistic matter (neutrinos) changes the shape of these regions of overdensity

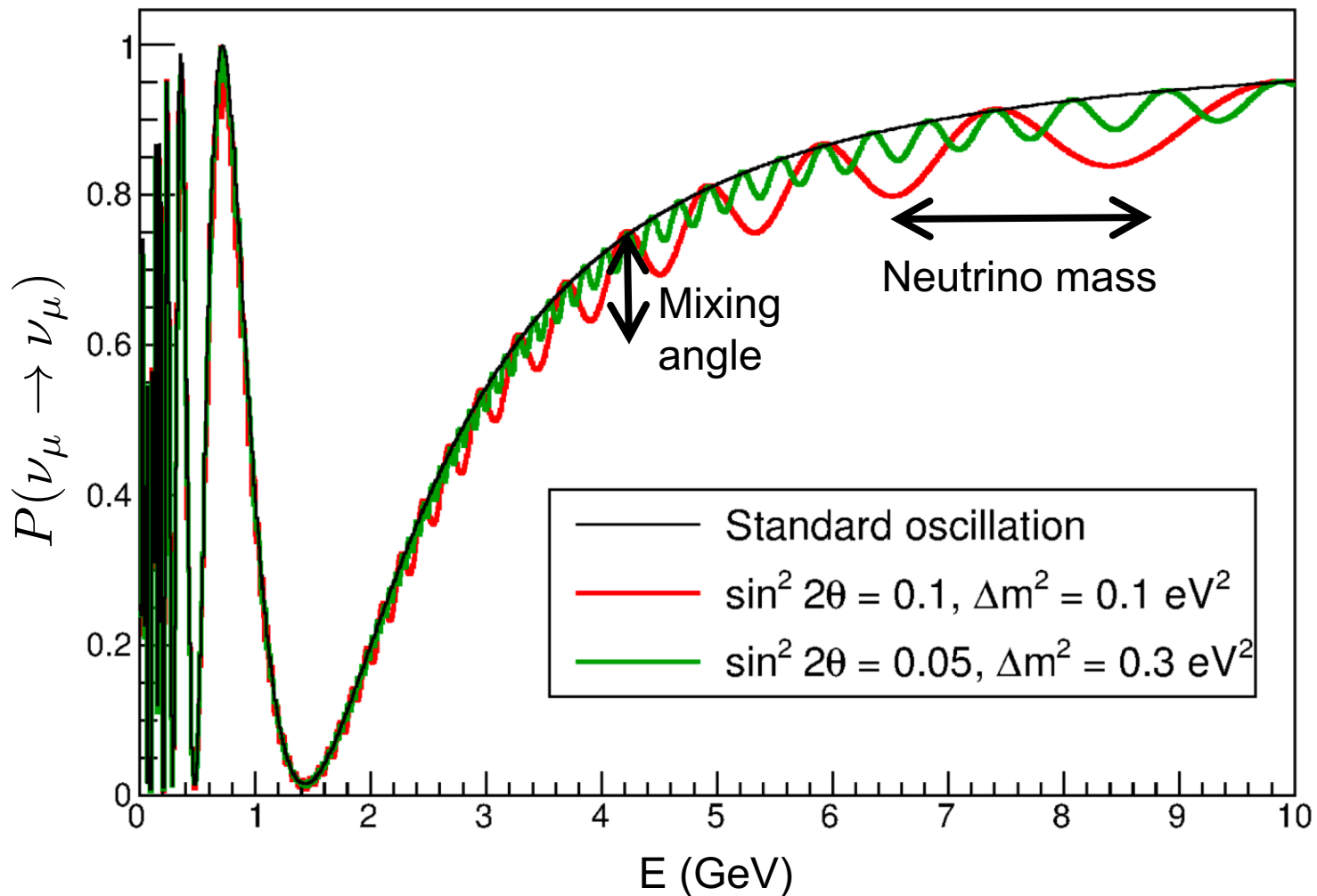
# Sterile neutrinos and the CMB



The ways in which neutrinos impact the CMB power spectrum are hugely complicated

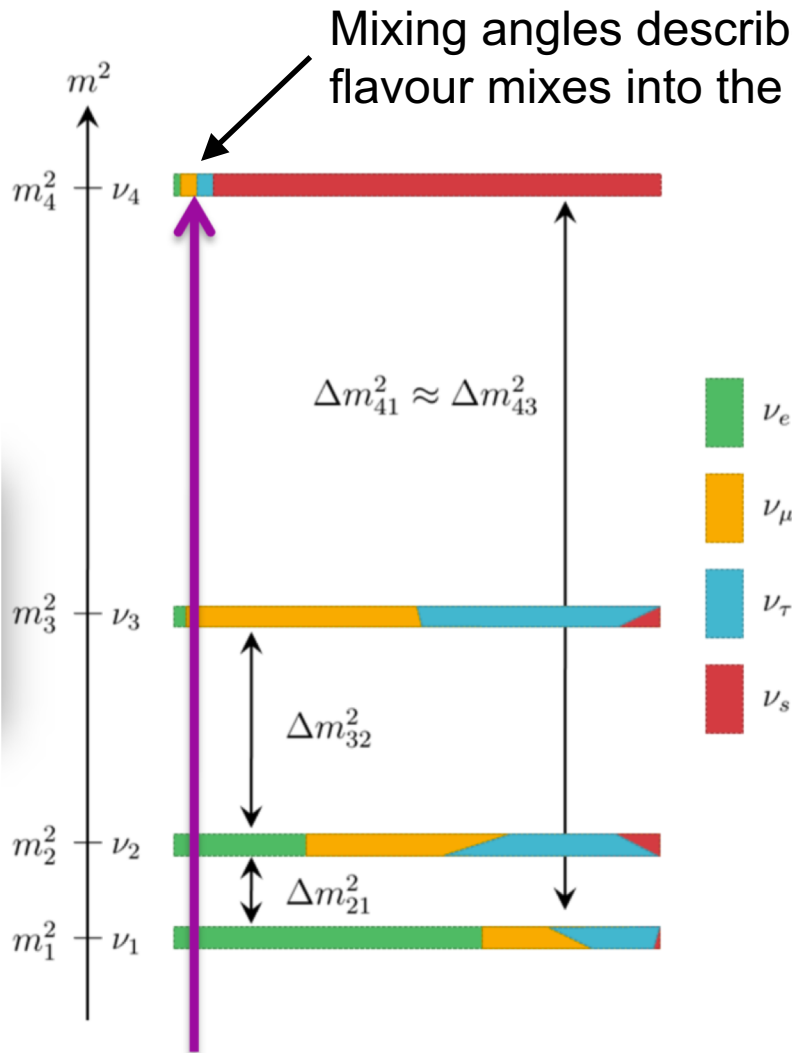
- But changes in  $m_{\text{eff}}$  and  $N_{\text{eff}}$  alter the positions and amplitudes of the peaks

# Sterile neutrinos and neutrino oscillations





# Relating particle physics to cosmology



$\theta_{14}$ : electron flavour  
 $\theta_{24}$ : muon flavour  
 $\theta_{34}$ : tau flavour

➤ Mass splitting relates fairly easily to  $m_{\text{eff}}$

$$m_{\text{eff}} = \sum_i m_i$$

➤ But what about the mixing angles and  $N_{\text{eff}}$ ?

# Decoupling of a sterile neutrino

S. Hannestad *et al.*, *Cosmol Astropar. Phys.* **2012**, 025 (2012), arXiv:1204.5861

K. Enqvist *et al.*, *Nucl. Phys. B* **373**, 498 (1992)

Typically work in a ‘two-flavour’ model

$$\nu_a = \cos \theta_s \nu_1 - \sin \theta_s \nu_2 \quad (\text{active flavour})$$

$$\nu_s = \sin \theta_s \nu_1 + \cos \theta_s \nu_2 \quad (\text{sterile flavour})$$

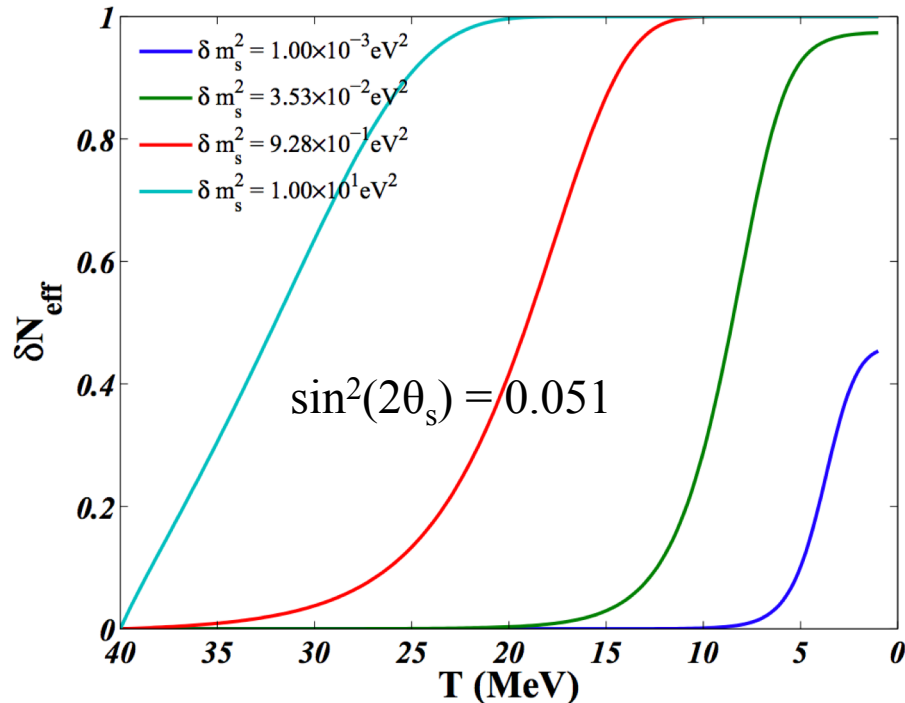
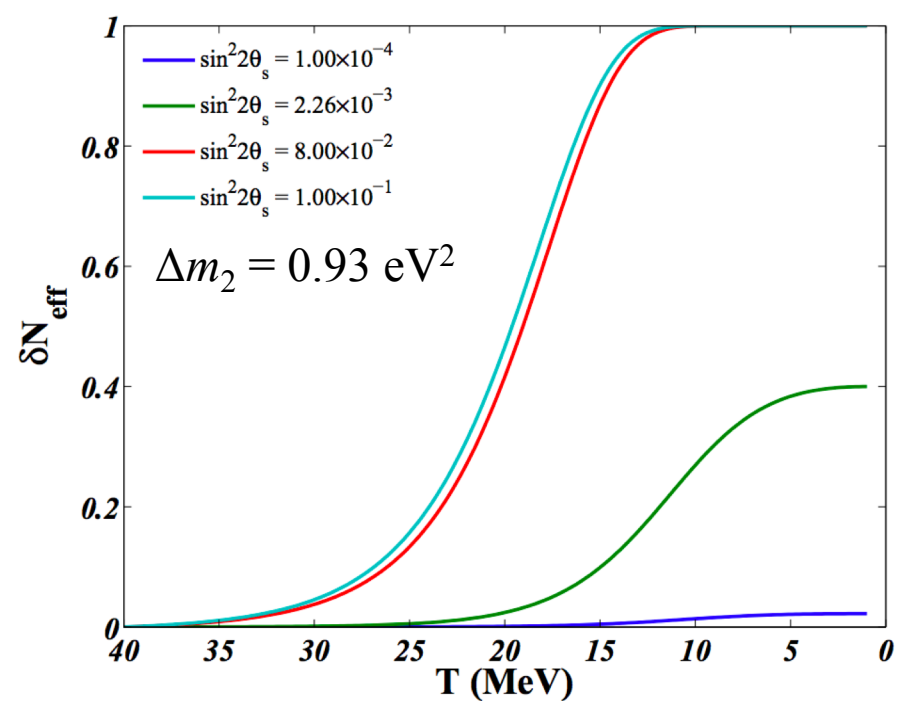
- $\theta_s$  is  $\theta_{14}$ ,  $\theta_{24}$  or  $\theta_{34}$ , depending on which active flavour we allow to mix into the new mass state

Hannestad and Enqvist papers numerically solve the quantum-kinetic equations

Define  $\delta N_{\text{eff}}$

- Additional relativistic degrees of freedom, beyond 3.046, introduced by the sterile neutrino
- The size of the mixing angle defines how strongly the sterile state couples to the Fermi-Dirac distribution before decoupling
- For a small mixing angle, the sterile neutrino does not produce an entire extra degree of freedom:  $\delta N_{\text{eff}} < 1$

# Decoupling of a sterile neutrino



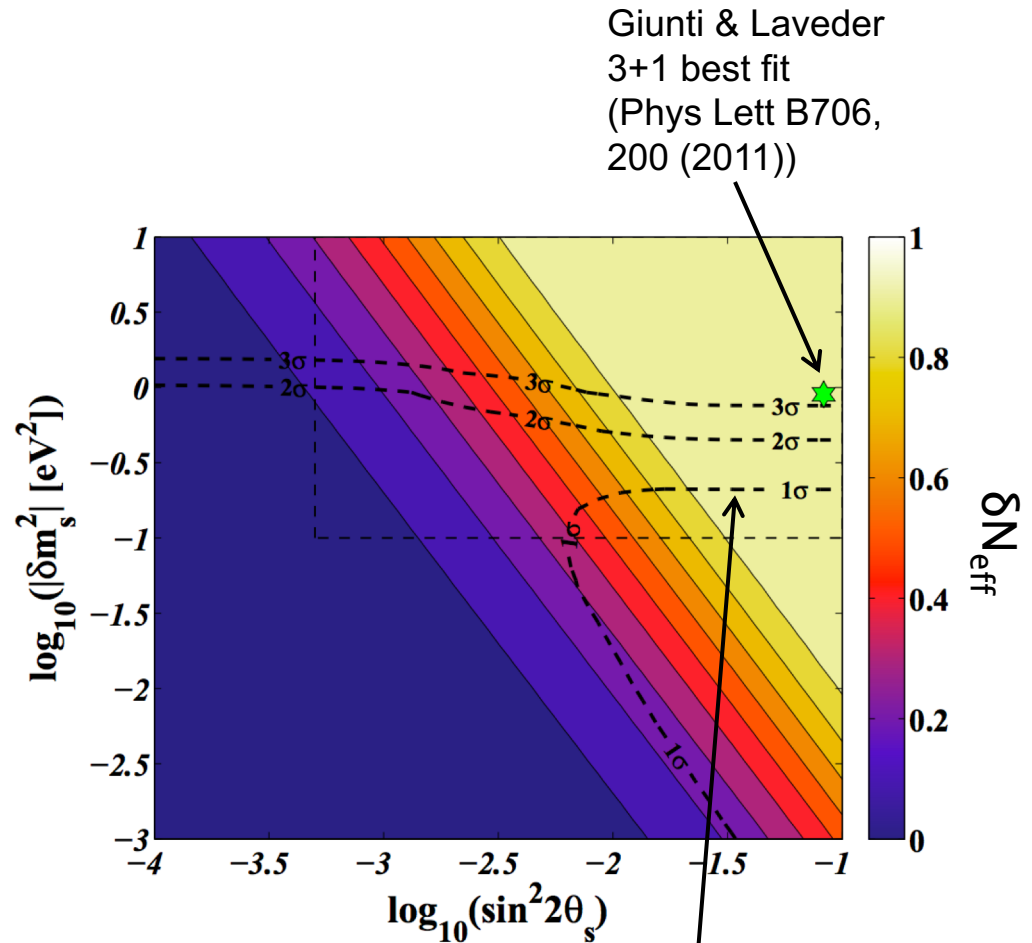
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# Relating particle physics to cosmology

The Hannestad *et al.* work allows us to relate our three parameters

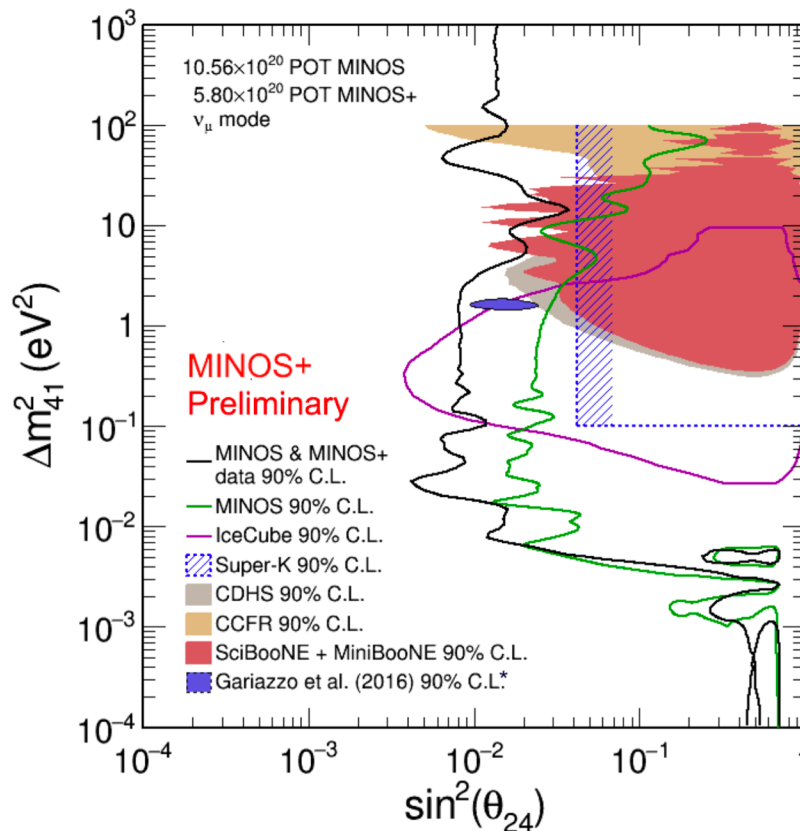
- $\delta m^2_s$ ,  $\sin^2(2\theta_s)$  and  $\delta N_{\text{eff}}$



# Muon neutrino measurements

MINOS result: Phys. Rev. Lett. **117**,  
151803 (2016) (not the most recent)

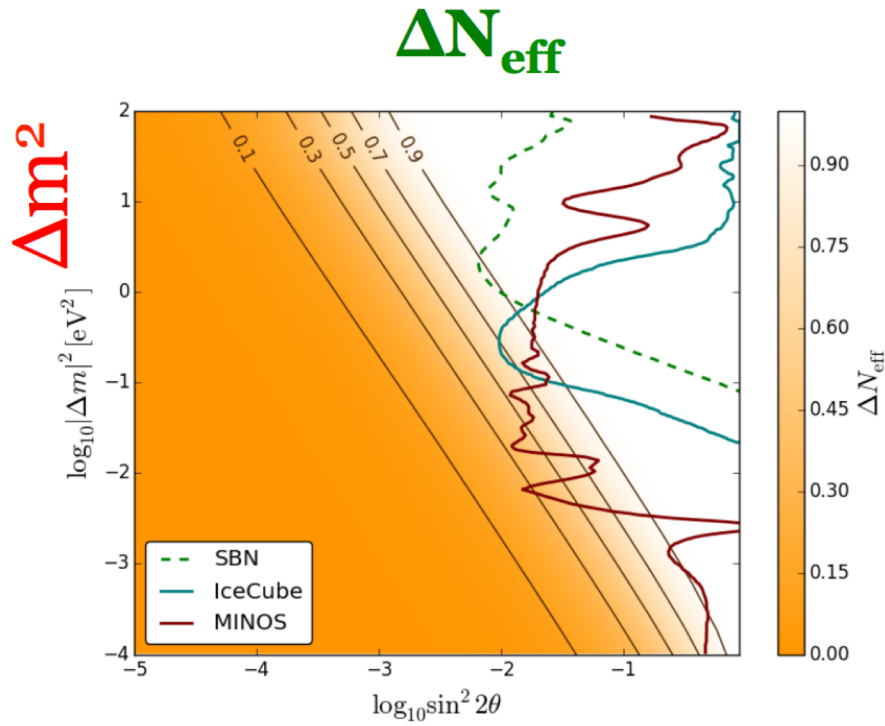
IceCube result: Phys. Rev. Lett. **117**,  
071801 (2016)



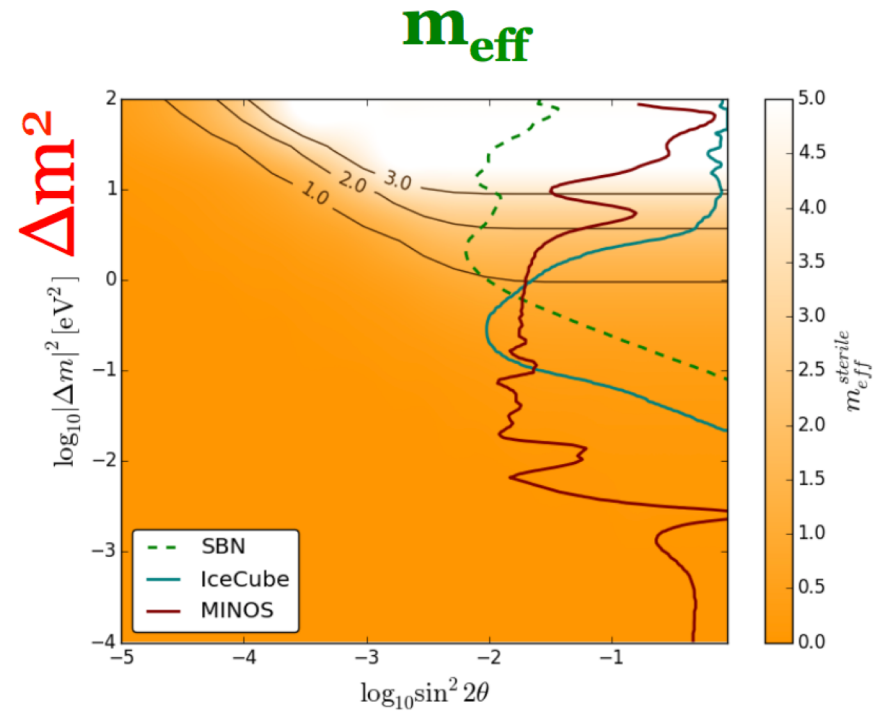
Assume  $\theta_{14} = \theta_{34} = 0$ , and leave  $\theta_{24}$  free

- Only muon flavour mixes with the fourth mass state
- Allows comparison with the MINOS and IceCube  $\nu_{\mu}$ -disappearance limits

# Particle physics $\rightarrow$ cosmology



$\sin^2 2\theta$



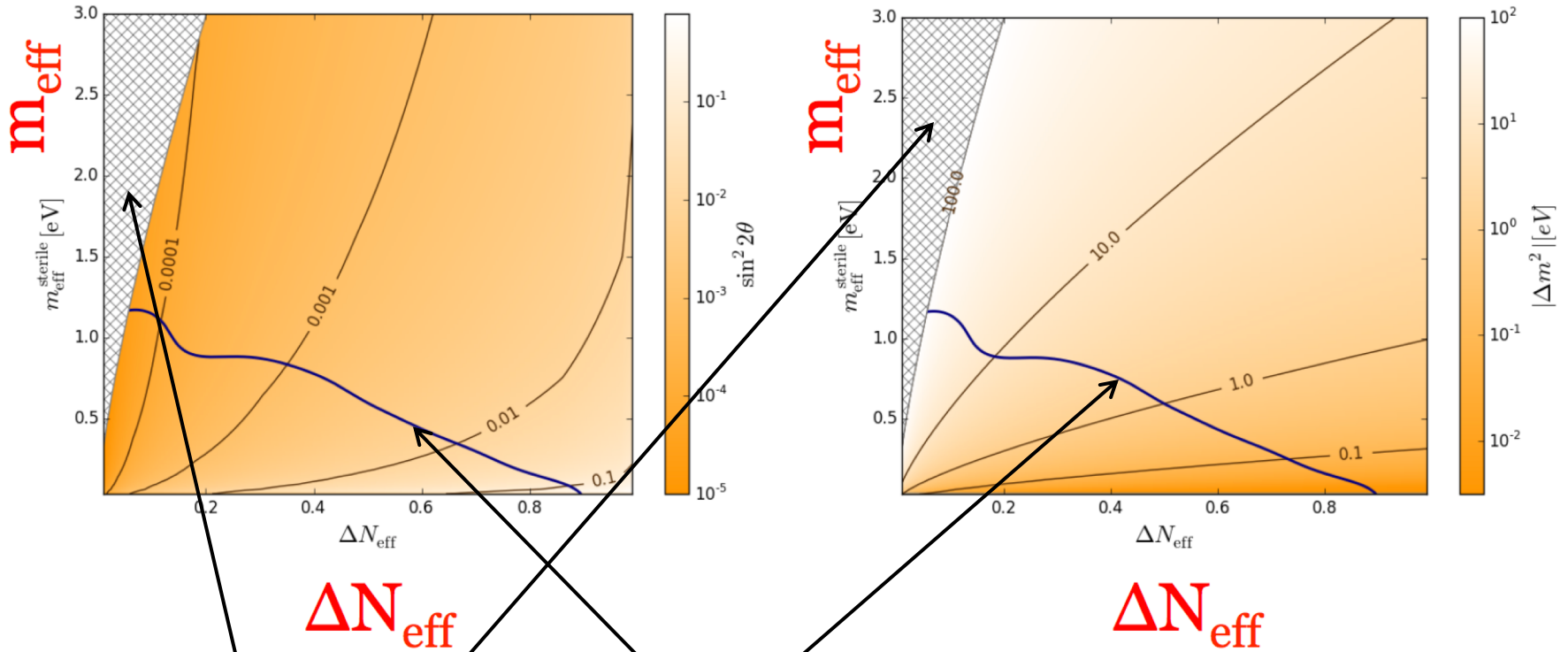
$\sin^2 2\theta$

S. Bridle *et al.*, Phys. Lett. **B764**, 322 (2017)

# Cosmology → particle physics

$\sin^2 2\theta$

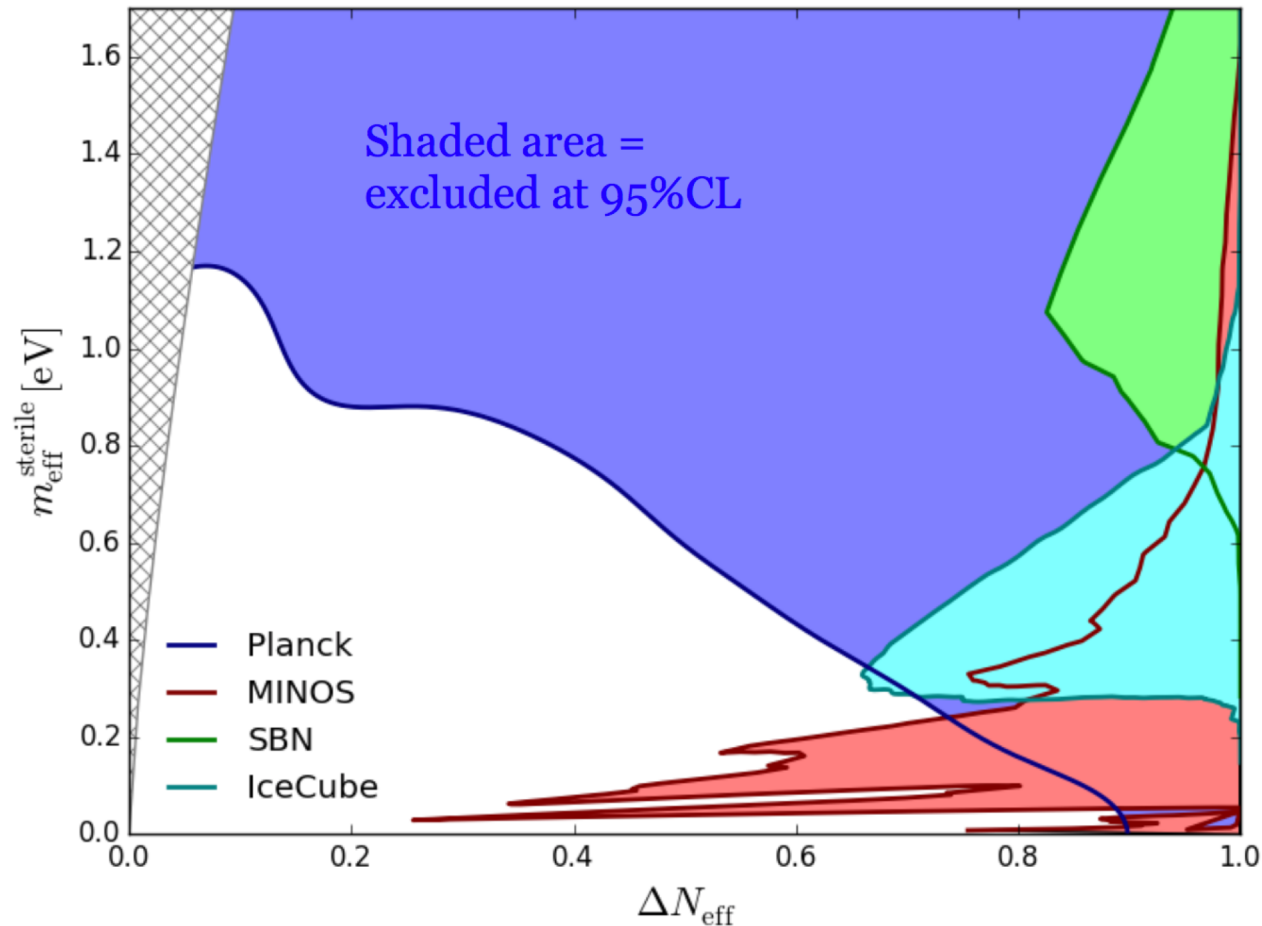
$\Delta m^2$



Plank 95% C.L. exclusion contour

Plank prior assumes  $m_4 < 10 \text{ eV}$  S. Bridle *et al.*, Phys. Lett. **B764**, 322 (2017)

# Cosmology space

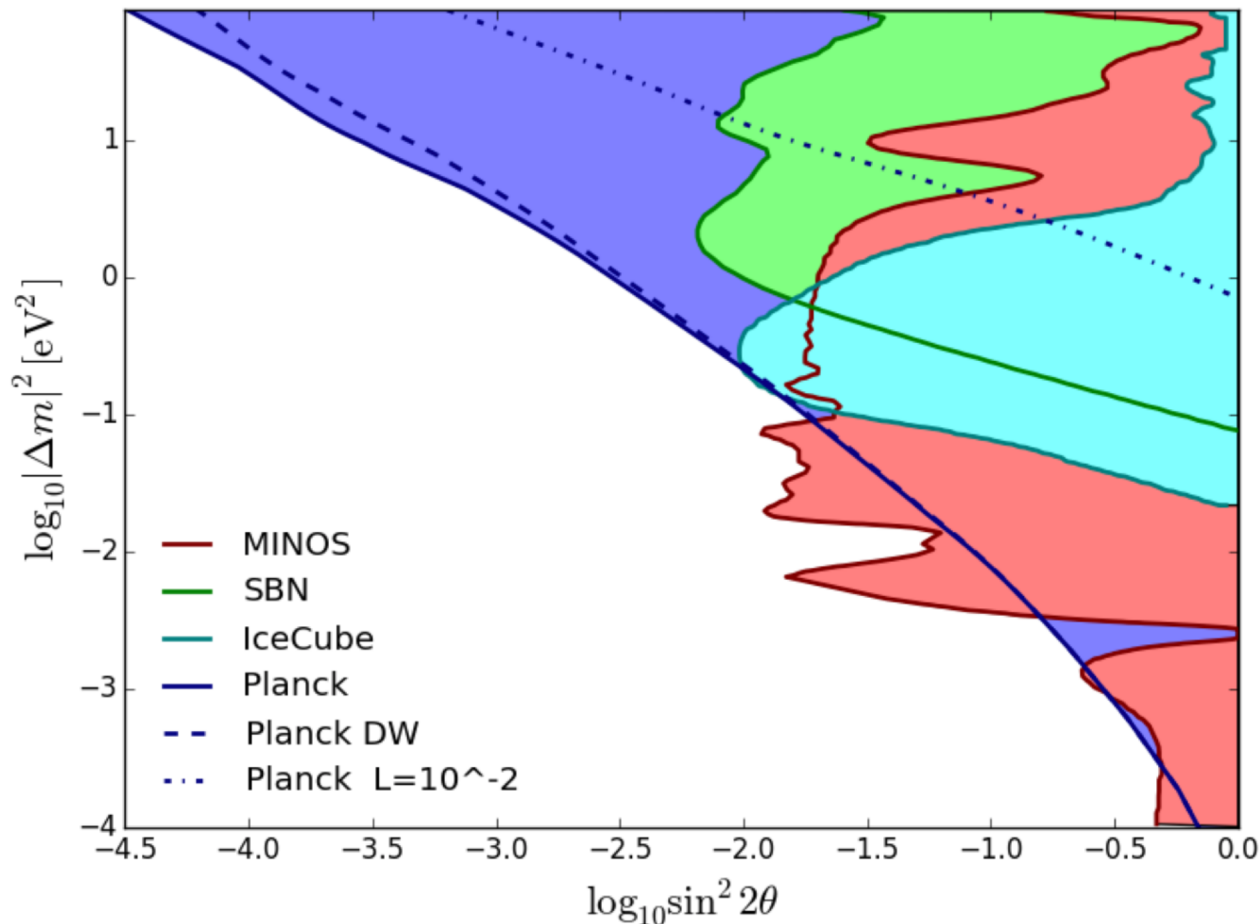


Cosmological limits are weakest at low masses, where MINOS+ becomes stronger

- And remember: new MINOS+ limits now available



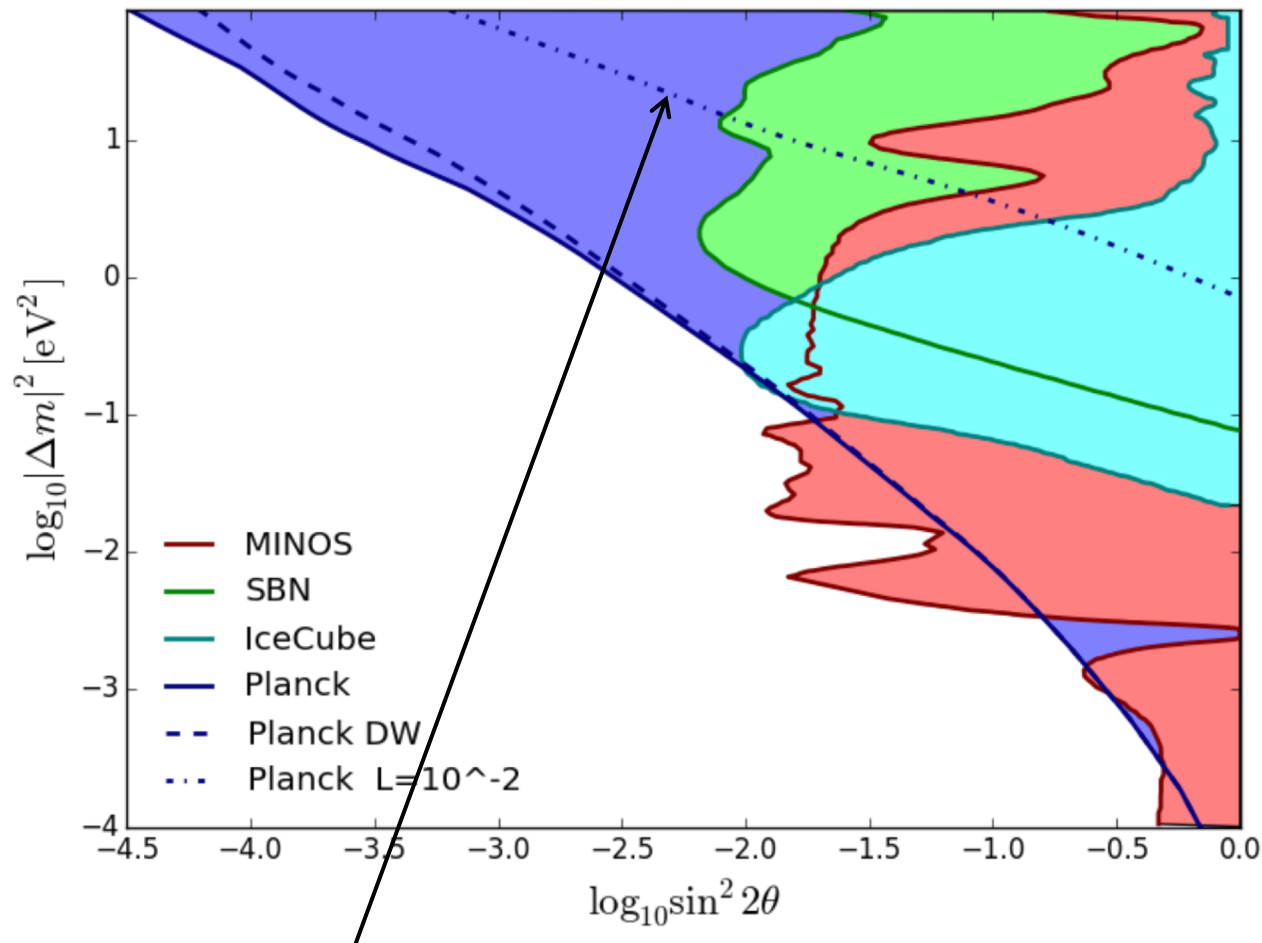
# Neutrino physics space



Cosmological limits are weakest at low masses, where MINOS+ becomes stronger

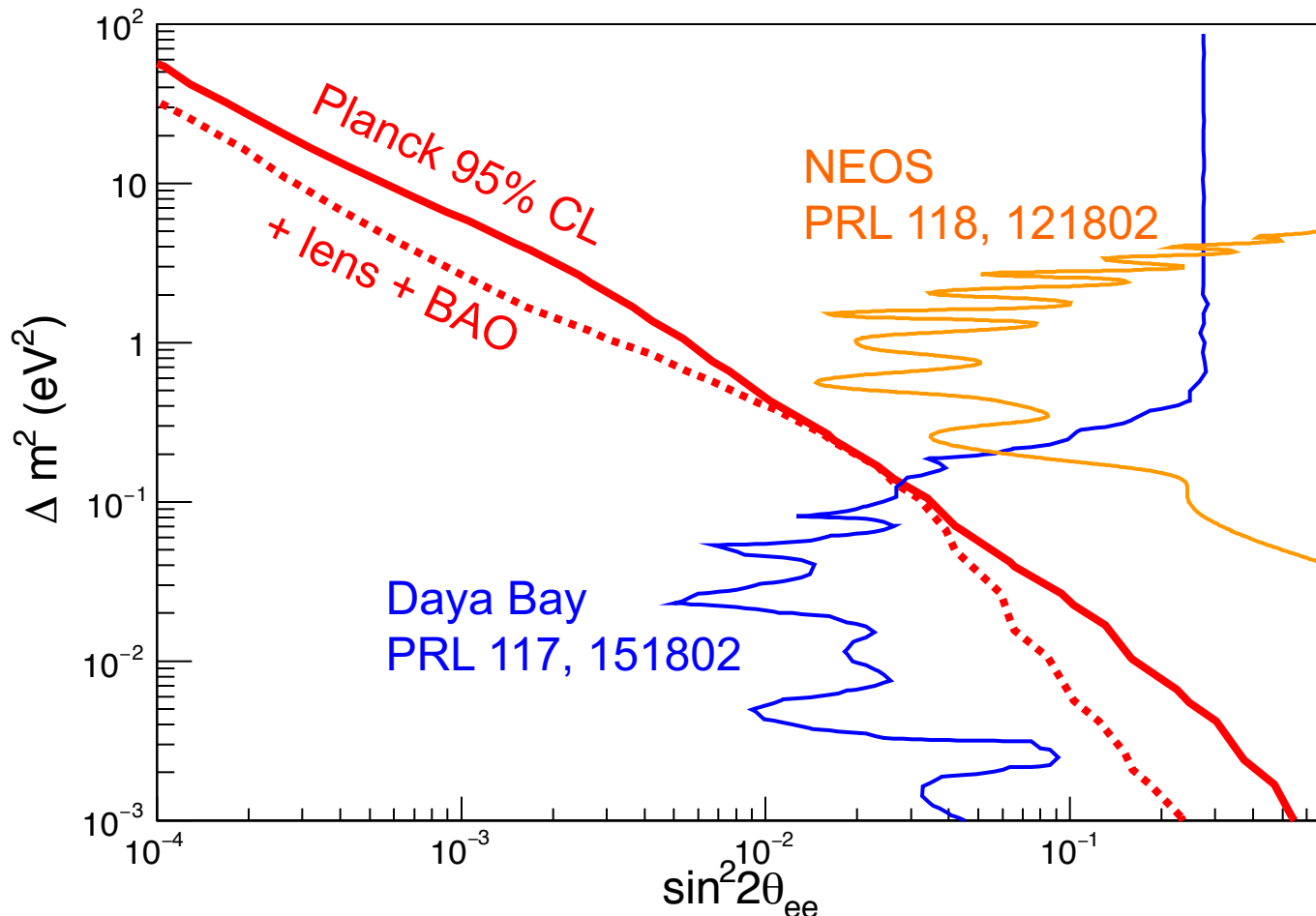
- And remember: new MINOS+ limits now available

# Model dependence



Adding in a neutrino–antineutrino asymmetry at the extreme of possibilities (i.e. a chemical potential)

# Electron-antineutrino disappearance limits

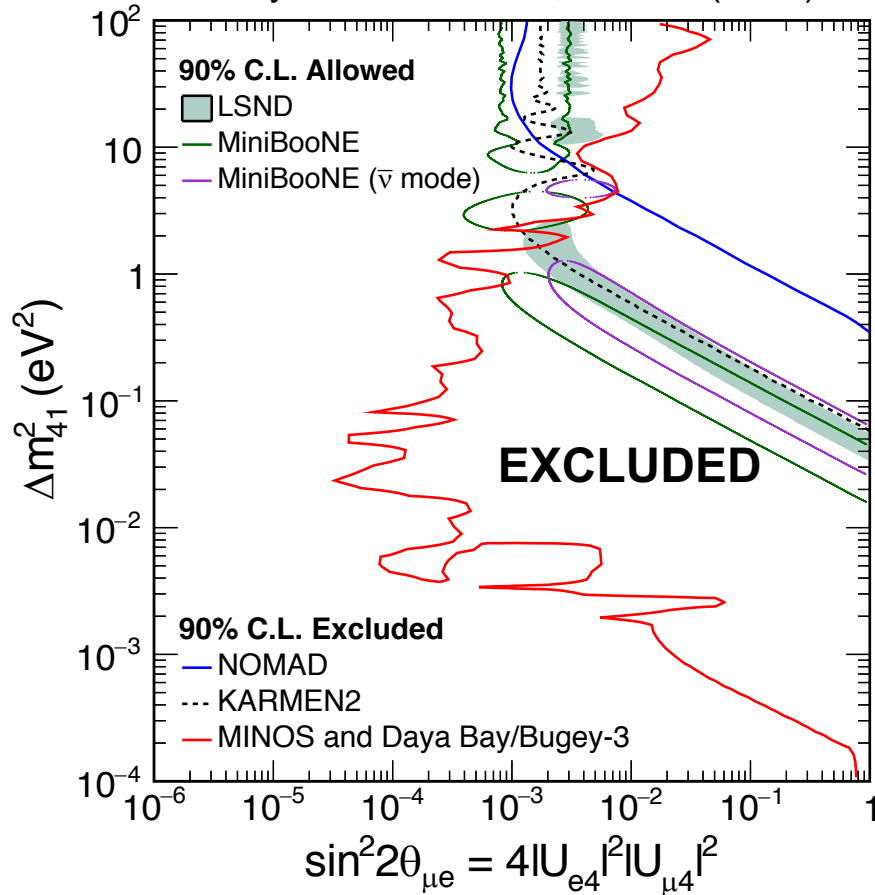


Assume  $\theta_{24} = \theta_{34} = 0$ , and leave  $\theta_{14}$  free

- Only electron flavour mixes with the fourth mass state
- Allows comparison with reactor limits

# Comparing to appearance results

Phys. Rev. Lett. **117**, 151803 (2016)



The LSND and MiniBooNE hints come from the channel  $\nu_{\mu} \rightarrow \nu_e$  (and the CP conjugate)

This requires both muon and electron flavour to mix with the fourth mass state

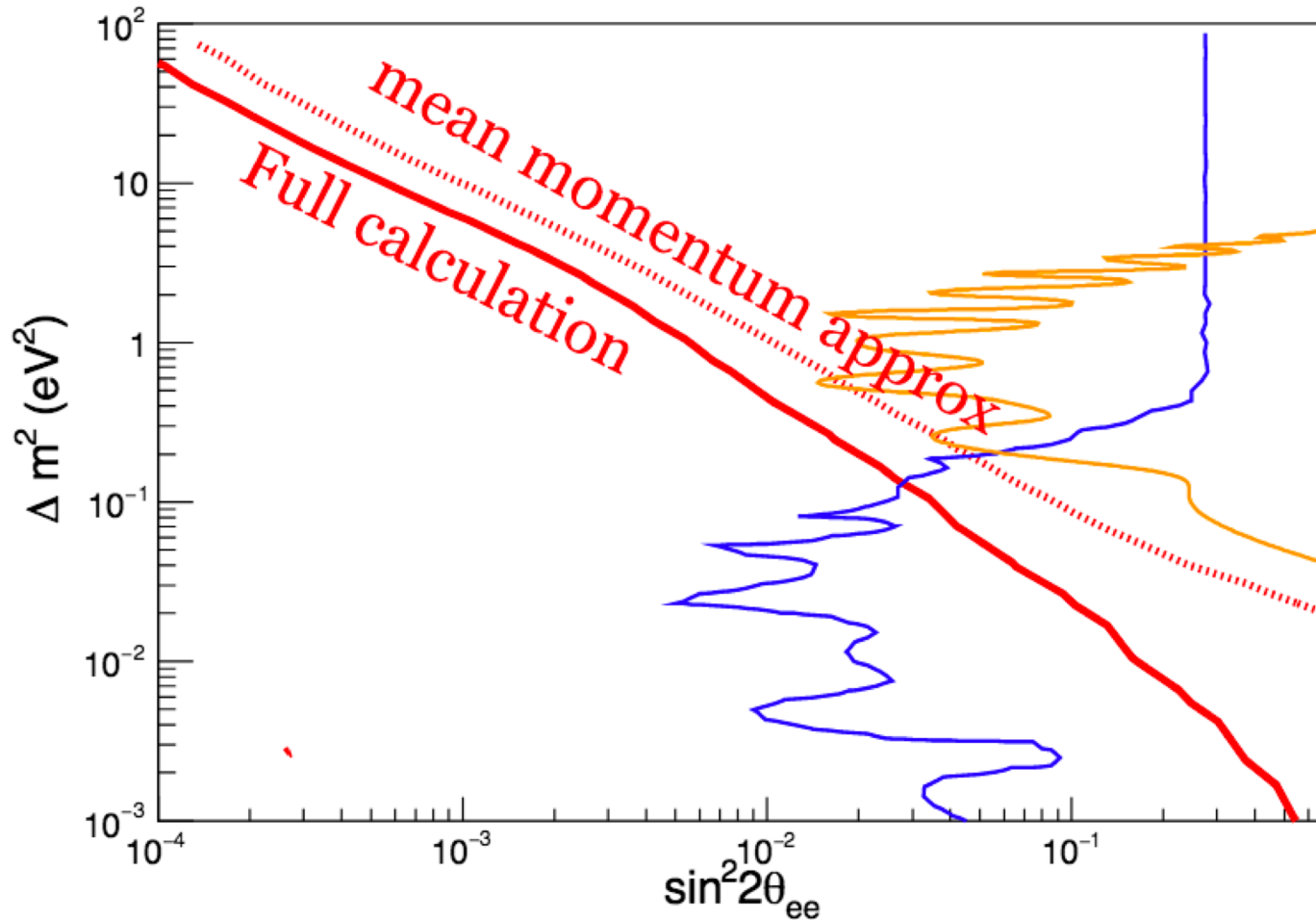
- $\theta_{24}$  and  $\theta_{14}$  must be free parameters

A. Mirizzi *et al.* Phys. Rev. **D86**, 053009 (2012) provides a prescription for decoupling the neutrino sea and calculating  $\delta N_{\text{eff}}$

But requires an approximation

- Instead of a Fermi-Dirac distribution, assume all neutrinos have the mean momentum

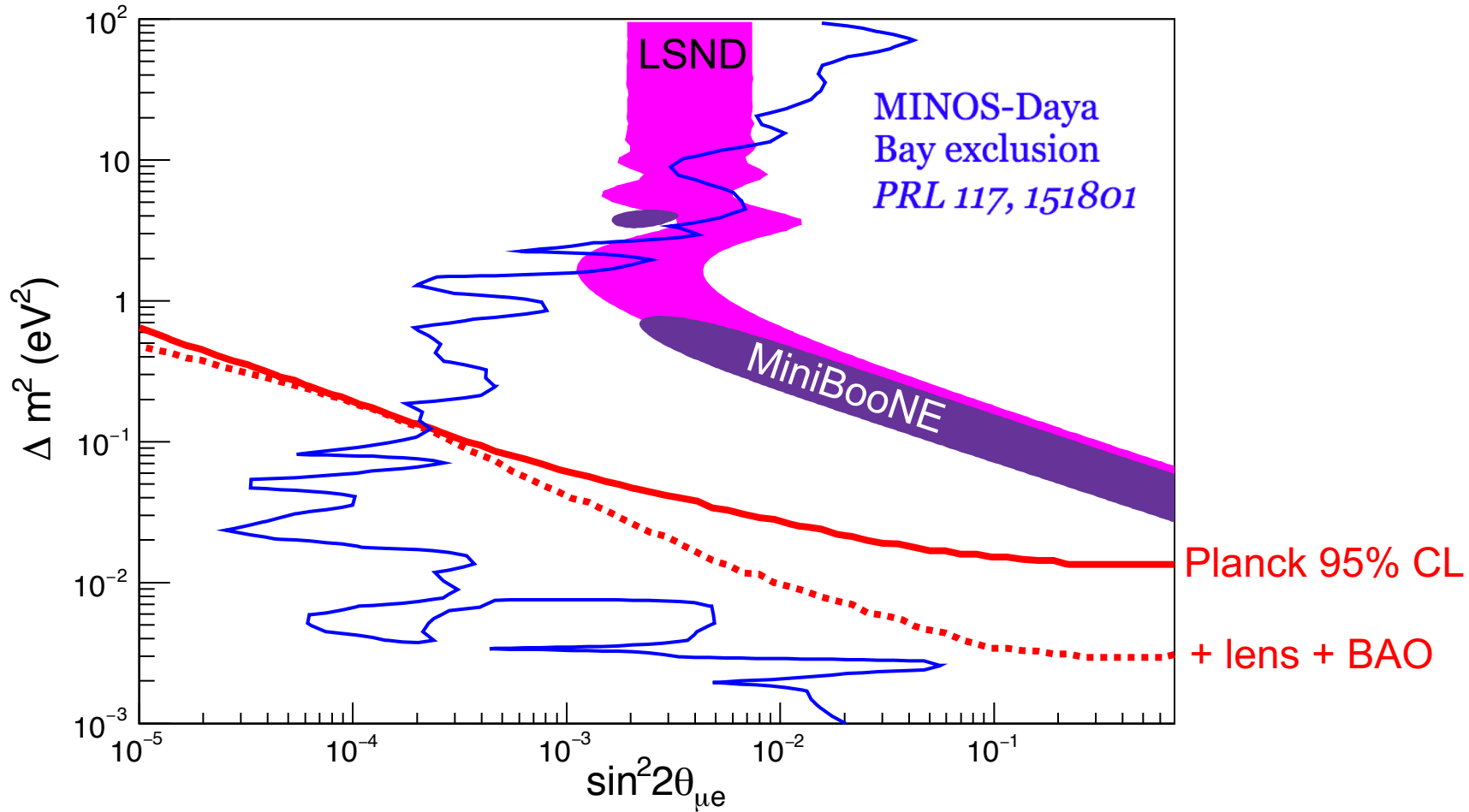
# Moving to three-flavours



Test this mean-momentum approximation on the electron-neutrino-only case

- Where we can do the exact calculation

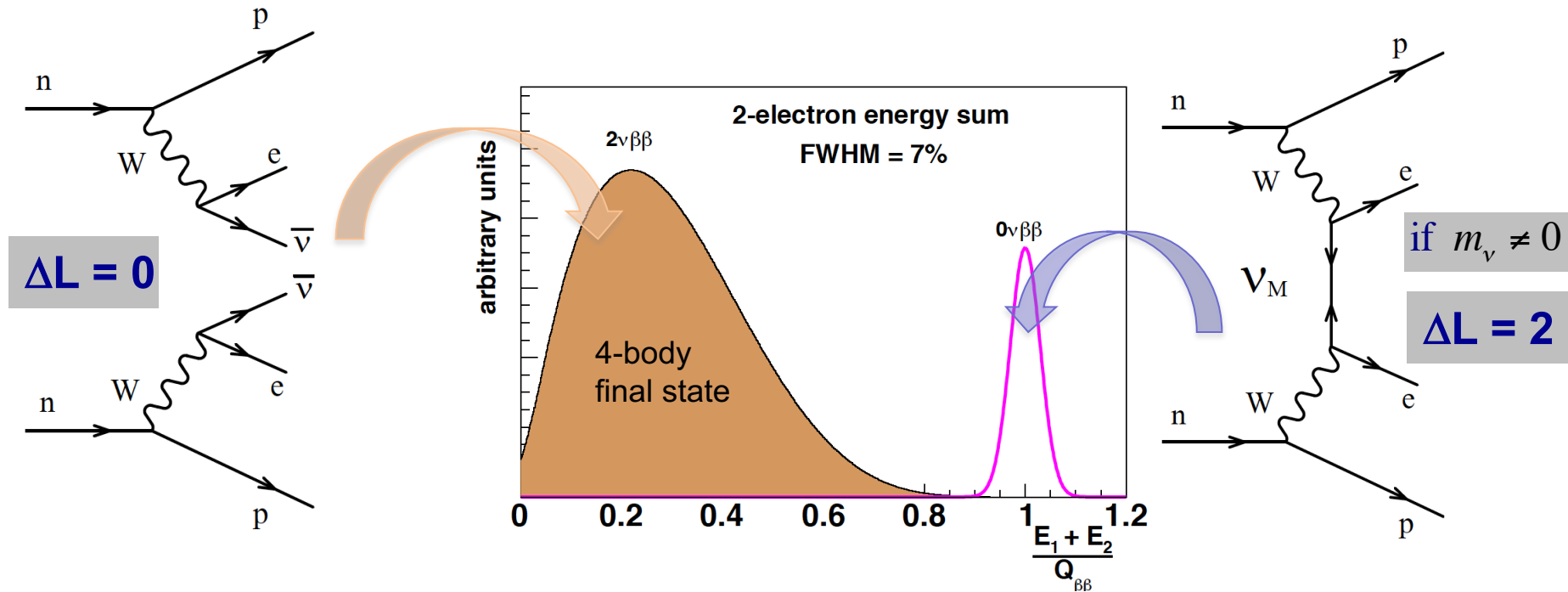
# $\nu_\mu \rightarrow \nu_e$ appearance limits



# An aside – Sterile neutrinos and $0\nu\beta\beta$

- P. Guzowski *et al.*, Phys. Rev. **D92**, 012002 (2015)

# Neutrinoless double beta decay



$$\frac{1}{T_{1/2}^{0\nu}} = G_{0\nu}(Q_{\beta\beta}^5, Z) \cdot |M_{0\nu}|^2 \cdot \langle m_{\beta\beta} \rangle^2$$

Phase space

Nuclear matrix element

Effective neutrino mass



# The effective neutrino mass

$$|\langle m_{\beta\beta} \rangle|^2 = \left| |U_{e1}|^2 m_1 + e^{i\alpha_1} |U_{e2}|^2 m_2 + e^{i\alpha_2} |U_{e3}|^2 m_3 \right|^2$$

# The effective neutrino mass

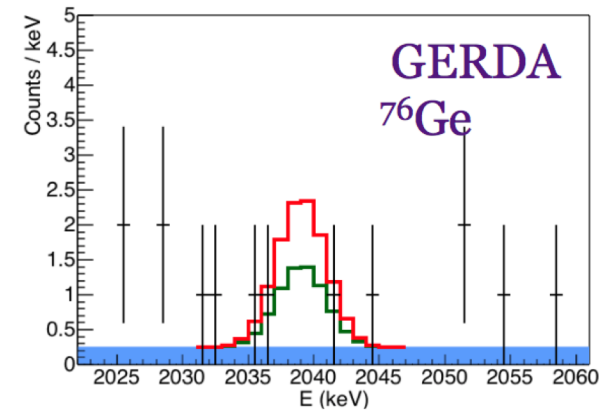
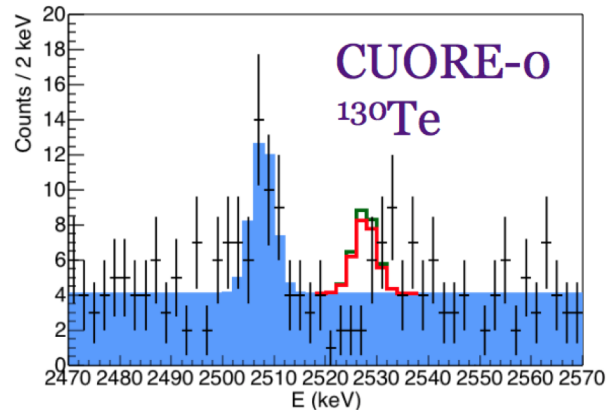
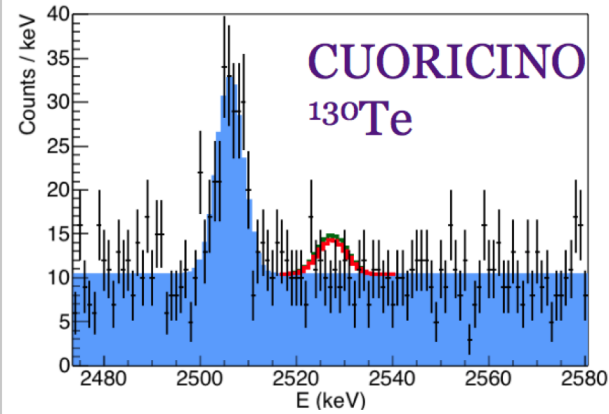
Now add a sterile neutrino:

$$\begin{aligned}
 |\langle m_{\beta\beta} \rangle|^2 = & \left| |U_{e1}|^2 m_1 + e^{i\alpha_1} |U_{e2}|^2 m_2 + e^{i\alpha_2} |U_{e3}|^2 m_3 \right. \\
 & \left. + \sin^2 \theta_{14} e^{i\alpha_3} m_4 \right|^2
 \end{aligned}$$

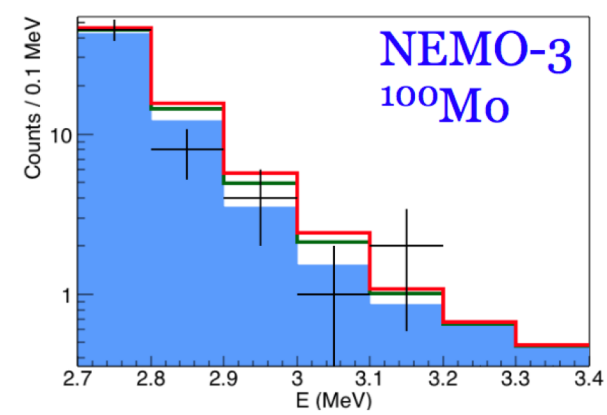
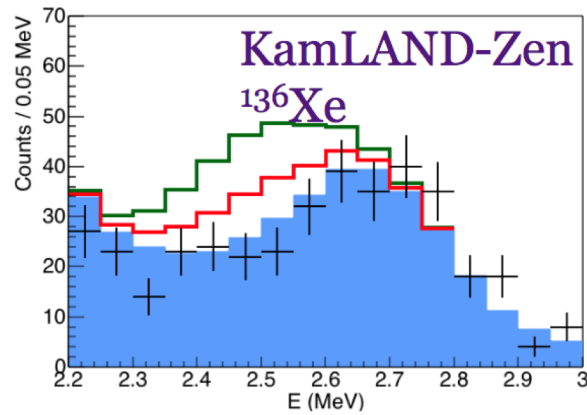
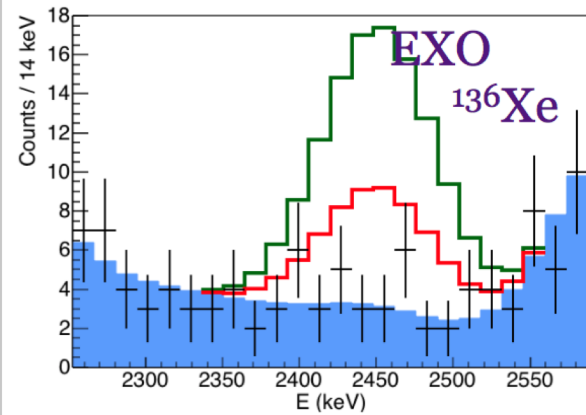
As  $\theta_{14}$  increases, the effective neutrino mass governing double beta decay increases

- Which would increase the  $0\nu\beta\beta$  decay rate

# Combine six experiments

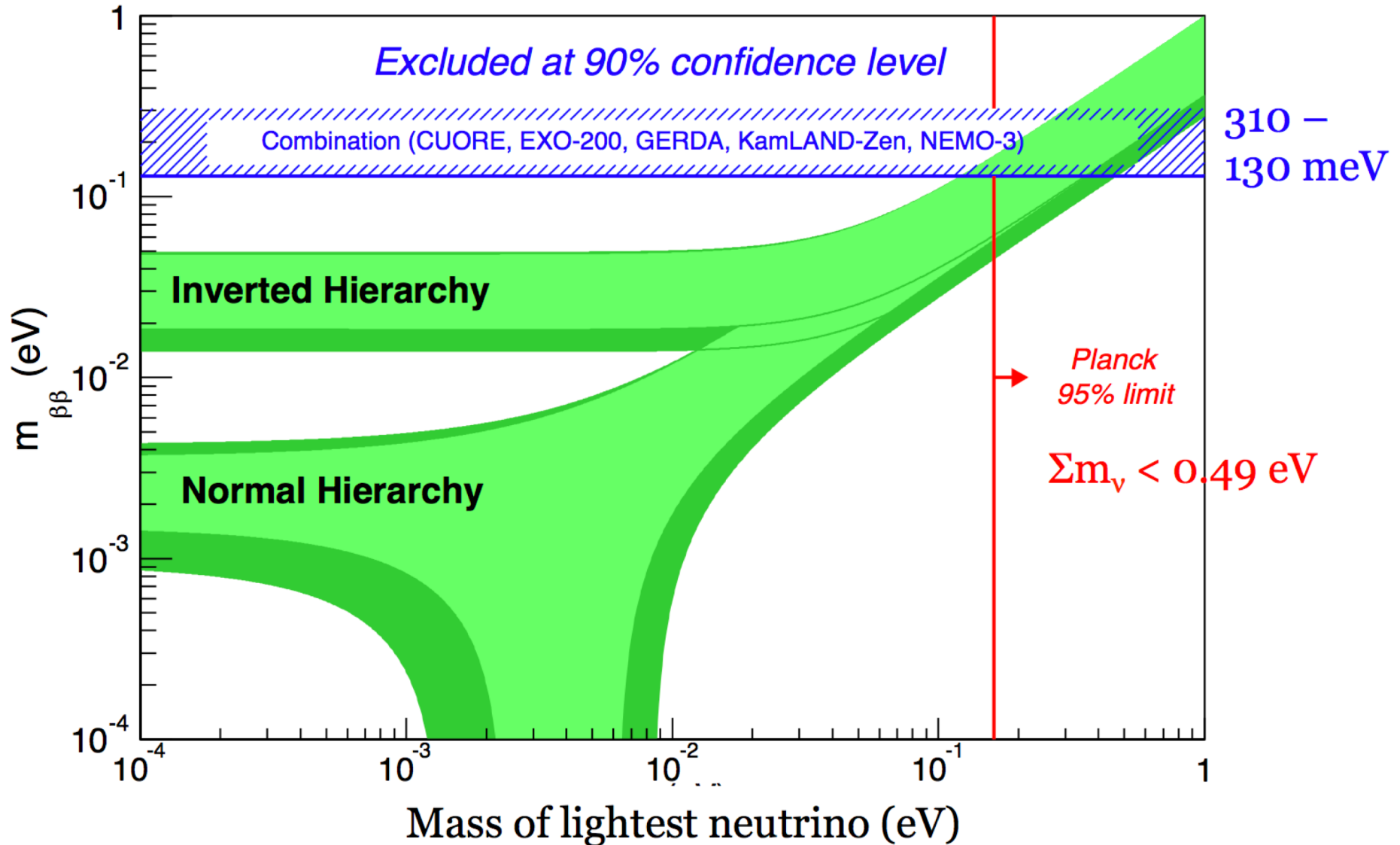


For the *GCM* or *QRPA* NMEs with  $m_{\beta\beta} = 400$  meV

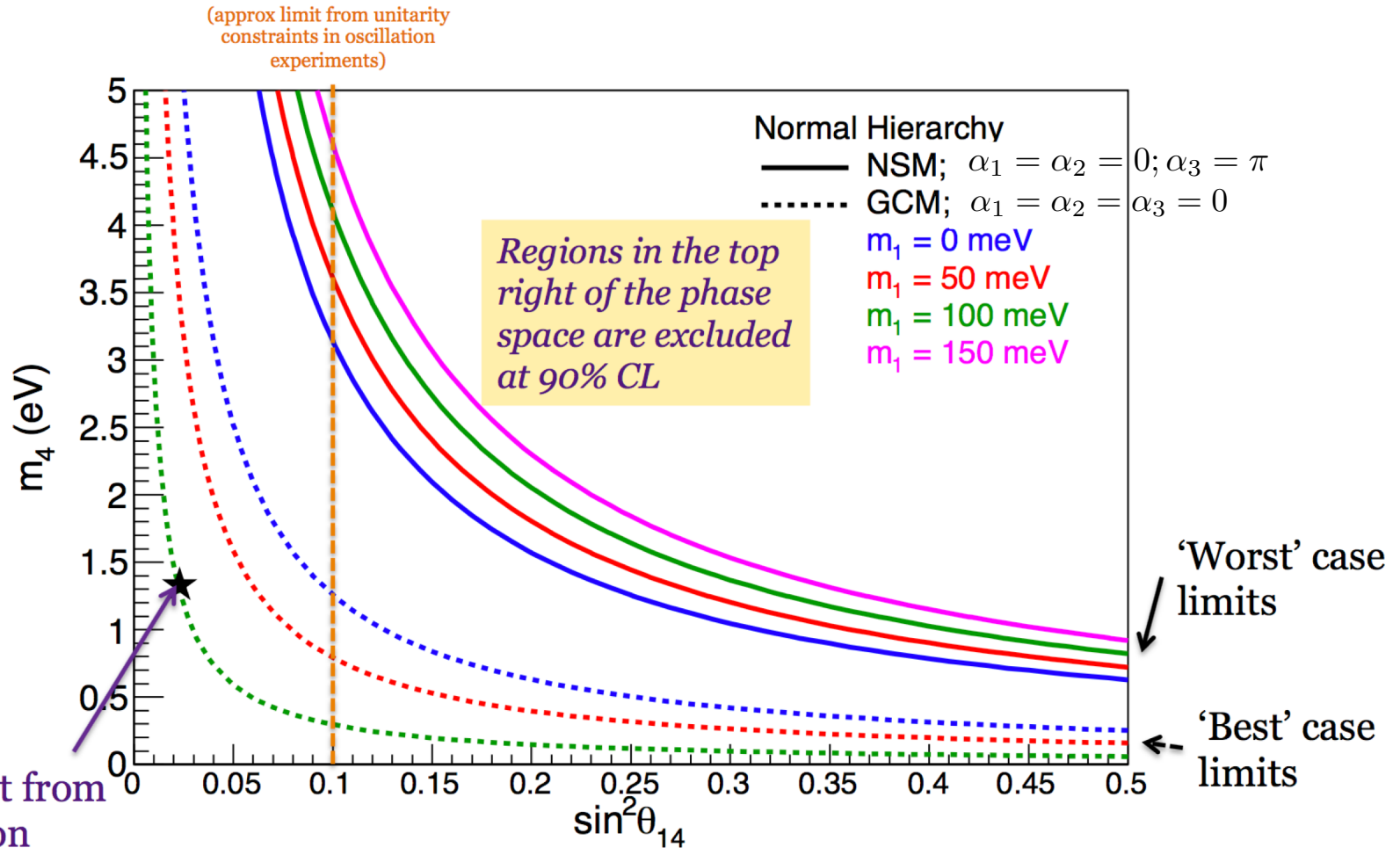


Guzowski, Barnes, Evans, Karagiorgi, McCabe, Söldner-Rembold,  
Phys. Rev. D **92**, 012002 (2016)

# Exclusion of 'regular' $0\nu\beta\beta$



# Excluding sterile Majorana neutrinos



Global fit from oscillation experiments  
*JHEP 05, 050 (2013)*

$$\Delta m_{41}^2 = 1.78 \text{ eV}^2, \quad \sin^2(2\theta_{14}) = 0.09$$

# Summary

Cosmological and particle physics searches for sterile neutrinos can be compared in the same parameter space

- Cosmological limits are strongest at mass splittings above  $\sim 0.1 \text{ eV}^2$  but are model-dependent
- MINOS+ limits are stronger at the lower mass splittings
- S. Bridle *et al.*, Phys. Lett. **B764**, 322 (2017)

If the neutrino is a Majorana particle, a sterile neutrino would impact  $0\nu\beta\beta$  decay rates

- Increasing the effective neutrino mass in a way that is dependent on  $\theta_{14}$
- The non-observation of  $0\nu\beta\beta$  can be used to place limits on sterile neutrino parameter space, assuming neutrinos are Majorana particles
- P. Guzowski *et al.*, Phys. Rev. **D92**, 012002 (2015)