Models and phenomenology of flavoured axions

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invisibles
neutrinos, dark matter & dark energy physics

Q: What, if anything, does flavour have to do with a solution to the strong CP problem?

- Model

  $U(1)$ flavour symmetries as Peccei-Quinn symmetries

  (to appear in JHEP) [1811.09637 [hep-ph]]

- Phenomenology

  Flavourful Axion Phenomenology

  JHEP 1808 (2018) 117 [1806.00660 [hep-ph]]

Recent developments:

[Celis, Fuentes-Martin, Serôdio '14] [Ahn '14 & '18] [FB, Chun, King '17 & '18]
[Ema, Hamaguchi, Moroi, Nakayama '16] [Calibbi, Goertz, Redigolo, Ziegler, Zupan '16]
[Linster, Ziegler '18] [Reig, Valle, Wilczek '18] [Alanne, Blasi, Goertz '18]
[Gavela, Houtz, Quilez, Del Rey, Sumensari '19]

- **Strong CP problem**
  a quirk of the Standard Model of particle physics. Is it really a problem?
- **Peccei-Quinn mechanism**
  (nearly) everyone's favourite solution to the above problem
- **Peccei-Quinn symmetry**
  a global $U(1)$ symmetry (like B or L) with certain characteristics; is spontaneously broken (like $SU(2)_L \times U(1)_Y$) by the vev of a new field.
- **axion**
  the Goldstone mode of the sp. br. symmetry. Gets a small mass from QCD (like pions).

A nice review on the strong CP problem: [Peccei, hep-ph/0607268]

The strong CP problem is of almost no consequence

[paraphrasing Michael Dine, talk 2015]

The strong *CP* problem is of almost no consequence

[paraphrasing Michael Dine, talk 2015]

Allowed term in QCD

$$\mathcal{L} \supset \bar{\theta} \frac{g^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad \bar{\theta} = \theta_{\mathrm{QCD}} + \arg\det M^u M^d$$

Values

- Measurement: neutron EDM [Pendlebury et al '15]

$$\bar{\theta} \lesssim 10^{-10}$$

- Naively: $\bar{\theta} \sim 1$
- Anthropically: $\bar{\theta} \sim 10^{-3}$ is fine [Dine, Draper '15]
- Exact CP ($\bar{\theta} = 0$) in QCD not technically necessary

Ingredients in a standard PQ solution

- Global $U(1)_{PQ}$ symmetry with QCD anomaly
- Complex scalar field $\varphi \to \langle\varphi\rangle$ which breaks $U(1)_{PQ}$

Archetypal "invisible axion" models

KSVZ

$$\mathcal{L} \supset \lambda\varphi\overline{Q}Q$$

- Add: heavy quarks Q
- Axion-ψ_{SM} coupling: loop level

DFSZ

$$\mathcal{L} \supset \lambda\varphi^2 H_u H_d$$

- Add: second Higgs doublet
- Axion-ψ_{SM} coupling: tree level

DFSZ Lagrangian

$$\mathcal{L} \sim \lambda_\phi \varphi^2 H_u H_d + Y_{ij}^{(u)} \overline{Q}_i u_j H_u + Y_{ij}^{(d)} \overline{Q}_i d_j H_d$$

Canonically, quark charges are generation-independent

- $\mathcal{X}(Q_i) = \mathcal{X}_Q$, etc
- Yukawa matrices $Y_{ij}^{u,d}$ full (no texture zeroes)
- Axion pheno dominated by $g_{a\gamma}$, g_{aN}, g_{ae}

However, universal quark $U(1)$ charges are not necessary for the PQ solution to work.

Generation-dependent PQ symmetry
$\Leftrightarrow$
flavour-dependent axion

More generally

Generation-sensitive symmetries
$\Leftrightarrow$
flavour-dependent interactions

This is also the basis for models of SM Yukawa couplings: symmetries control Yukawa/mass textures.

A minimal $U(1)$ model of quark flavour

[FB, Di Luzio, Mescia, Nardi '18]

Assume

- 2HDM with $Y(H_{1,2}) = -1/2$
- Global $U(1)$ symmetry acting on quarks and Higgs
- Quark $U(1)$ charges can be generation-dependent

Define $U(1)$ charges $\mathcal{X}$

$$\begin{aligned} \mathcal{X}(H_{1,2}) &\equiv \mathcal{X}_{1,2} \\ \mathcal{X}(Q) &\equiv \{-x, -y, 0\} \\ \mathcal{X}(u) &\equiv \{a, b, c\} \\ \mathcal{X}(d) &\equiv \{m, n, p\} \end{aligned}$$

We may write combined charges of quark bilinears as matrices:

$$\mathcal{X}_{\overline{Q}u} = \begin{pmatrix} a+x & b+x & c+x \\ a+y & b+y & c+y \\ a & b & c \end{pmatrix}, \quad \mathcal{X}_{\overline{Q}d} = \begin{pmatrix} m+x & n+x & p+x \\ m+y & n+y & p+y \\ m & n & p \end{pmatrix}$$

If

$$(\mathcal{X}_{\overline{Q}u})_{ij} + \mathcal{X}_{1\,\mathrm{or}\,2} = 0 \quad \mathrm{or} \quad (\mathcal{X}_{\overline{Q}d})_{ij} - \mathcal{X}_{1\,\mathrm{or}\,2} = 0$$

the corresponding Yukawa coupling

$$\mathcal{L} \supset H_{1\,\mathrm{or}\,2}\overline{Q}_i u_j \ \mathrm{or}\ \tilde{H}_{1\,\mathrm{or}\,2}\overline{Q}_i d_j$$

is allowed. Conversely, if $\cdots \neq 0$, Yukawa matrix has texture zero.

What is the minimal set of non-zero Yukawa operators compatible with this $U(1)$ symmetry?

Conditions for a physically viable Yukawa sector

1. $U(1)$ charge consistency
2. Non-zero quark masses

$$\det M_u \neq 0, \quad \det M_d \neq 0$$

3. Non-vanishing Jarlskog invariant (i.e. a “full” CKM matrix)

$$J \propto \mathcal{D} \equiv \det[M_d M_d^\dagger, M_u M_u^\dagger] \neq 0$$

With 9 quark fields, we can perform 8 relative phase redefinitions to remove phases in M_u, M_d. We must have $8+1=9$ non-zero terms across $M_u \oplus M_d$ to have CP violation.

Structures that don't work

We need 9 non-zero Yukawa couplings: $M_n \oplus M_{9-n}$

Ex 1: $M_1 \oplus M_8$

$$M_u = M_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_d = M_8 = \begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$$

$\Rightarrow \det M_u = 0$

Ex 2: $M_3 \oplus M_6$

$$M_u = M_3 = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_d = M_6 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & \times & \times \end{pmatrix}$$

$\Rightarrow$ impossible to write consistent set of quark charges

If at all, only $M_4 \oplus M_5$ structures are compatible with physics!
Up to row/column permutations there is only one M_4 texture:

$$\begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

Ex 3: $M_4 \oplus M_5$

$$M_u = M_4 = \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad M_d = M_5 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$\Rightarrow J \propto \sin\theta_{13} = \sin\theta_{23} = 0$

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There are only 2 viable structures, both like $M_4 \oplus M_5$

$$\mathcal{T}_1 = \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \oplus \begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}$$

$$\mathcal{T}_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & 0 \end{pmatrix} \oplus \begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}$$

- Equivalent SM physics for any column or (simultaneous) row permutations, i.e. by redefinitions of quark fields
- One quark has no mixing: it is "sequestered"
- New Physics depends on sequestered quark $\Rightarrow 2 \times 6$ physically distinct textures

- It is possible to completely reconstruct the Yukawa matrices in terms of measured observables:
  - 9 (real) + 1 (phase) Yukawa parameters
  - 6 quark masses + 3 CKM mixing angles + 1 CP phase
  - At high scales ($\mu \sim 10^{12}$ GeV):

Observable	Value	Observable	Value
m_u /MeV	$0.61^{+0.19}_{-0.18}$	θ_{12}	0.22735 ± 0.00072
m_c /GeV	$0.281^{+0.02}_{-0.04}$	θ_{13}	0.00364 ± 0.00013
m_t /GeV	82.6 ± 1.4	θ_{23}	0.04208 ± 0.00064
m_d /MeV	1.27 ± 0.22	δ	1.208 ± 0.054
m_s /MeV	26^{+8}_{-5}		
m_b /GeV	$1.16^{+0.07}_{-0.02}$		

[Xing et al '11, Antusch, Maurer '13]

- Exact analytical expressions are possible, but ugly
- Solutions are stable under perturbations

The U(1) flavour symmetries are Peccei-Quinn symmetries!

- Anomaly

$$N = \frac{1}{2}\sum_i \left[\mathcal{X}(u) + \mathcal{X}(d) - 2\mathcal{X}(Q)\right]_i$$

- With normalization $\mathcal{X}_2 - \mathcal{X}_1 = 1$, we obtain

$$N(\mathcal{T}_1) = 1, \quad N(\mathcal{T}_2) = 1/2$$

- The Goldstone of the broken flavour $U(1)$ is an axion
- To be compatible with low-energy pheno, we make it *invisible*
  - $U(1)$ broken at high scale by new scalar ϕ
- Couplings are generation-dependent $\Rightarrow$ the axion is *flavoured*

Phenomenology

Axion mass comes from QCD, via mixing with the pion.

$$m_a = \frac{\sqrt{m_u m_d}}{(m_u + m_d)} \frac{m_\pi f_\pi}{f_a} \simeq 5.7\ \mu eV \times \left(\frac{10^{12}\ \mathrm{GeV}}{f_a} \right)$$

For precise calculation, see [Grilli, Hardy, Vega, Villadoro '16]

Axion-photon coupling

$$g_{a\gamma} = \frac{\alpha}{2\pi f_a} \left[\frac{E}{N} - 1.92 \right]$$

e.g. if $E/N = 8/3$ and $f_a \approx 10^{10}$ GeV,

$$g_{a\gamma} \approx 8.7 \times 10^{-14}\ GeV^{-1}$$

Axion couplings to fermions

$$\mathcal{L}_{af} = -\frac{\partial_\mu a}{2f_a} \sum_{f=u,d,e} \bar{f}_i \gamma^\mu (V^f_{ij} - A^f_{ij}\gamma_5) f_j ,$$

where $v_{PQ} = N_{DW} f_a = 2N f_a$ and

$$V^f = \frac{1}{2N}\left(U^\dagger_{Lf} x_{f_L} U_{Lf} + U^\dagger_{Rf} x_{f_R} U_{Rf}\right)$$
$$A^f = \frac{1}{2N}\left(U^\dagger_{Lf} x_{f_L} U_{Lf} - U^\dagger_{Rf} x_{f_R} U_{Rf}\right)$$

- $x_{f_L} = \mathrm{diag}(x_{f_{L1}}, x_{f_{L2}}, x_{f_{L3}})$, $x_{f_R} = \mathrm{diag}(x_{f_{R1}}, x_{f_{R2}}, x_{f_{R3}})$
- U_{Lf} and U_{Rf} are unitary matrices: $Y^f_{\mathrm{diag}} = U^\dagger_{Lf} Y^f U_{Rf}$
- $V_{\mathrm{CKM}} = U^\dagger_{Lu} U_{Ld}$

$$V^f = \frac{1}{2N}\left(U_{Lf}^\dagger x_{f_L} U_{Lf} + U_{Rf}^\dagger x_{f_R} U_{Rf}\right)$$
$$A^f = \frac{1}{2N}\left(U_{Lf}^\dagger x_{f_L} U_{Lf} - U_{Rf}^\dagger x_{f_R} U_{Rf}\right)$$

Special cases

1. All generations couple equally: $x_{f_L}, x_{f_R} \propto I_3$

$$\begin{array}{rcl} V^f & = & \frac{1}{2}(x_{f_L} + x_{f_R})\mathbb{I}_3 \\ A^f & = & \frac{1}{2}(x_{f_L} - x_{f_R})\mathbb{I}_3 \end{array} \Rightarrow \text{no flavour violation!}$$

2. Anomaly-free: $x_{f_L} = x_{f_R}$
   $\rightarrow$ no chiral anomaly ($N = 0$) $\rightarrow$ no PQ solution!

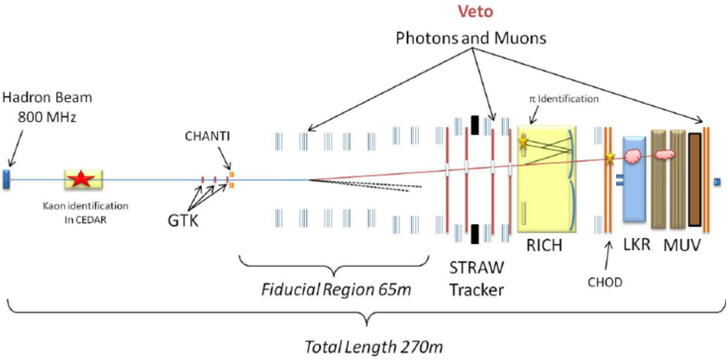
Decay: $P \to P' a$, where $P = (\bar{q}_P q')$, $P' = (\bar{q}_{P'} q')$.
Branching ratio

$$\mathrm{Br}(P \to P' a) = \frac{1}{16\pi\Gamma(P)} \frac{\left|V^f_{q_P q_{P'}}\right|^2}{(2f_a)^2} m_P^3 \left(1 - \frac{m_{P'}^2}{m_P^2}\right)^3 |f_+(0)|^2 \,,$$

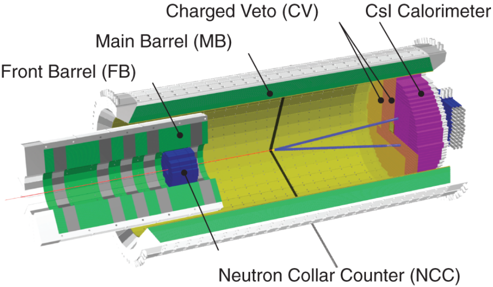
- $f_+(0)$ is a hadronic form factor
- Only unknown quantity is the ratio $|V^f|/f_a$
- Example: $K^+ \to \pi^+ a$ decay proceeds by $\bar{s} \to \bar{d} a$ with coupling strength $V^d_{sd} \equiv V^d_{21}$

Decay	$f_+(0)$
$K \to \pi$	1
$D \to \pi$	0.74(6)(4)
$D \to K$	0.78(5)(4)
$D_s \to K$	0.68(4)(3)
$B \to \pi$	0.27(7)(5)
$B \to K$	0.32(6)(6)
$B_s \to K$	0.23(5)(4)

- NA62 @ CERN SPS: $K^+ \to \pi^+ a$ ($K^+ \to \pi^+ \nu\bar{\nu}$)
  - Current status: one $\nu\bar{\nu}$ "event" [R. Marchevski at Moriond '18]

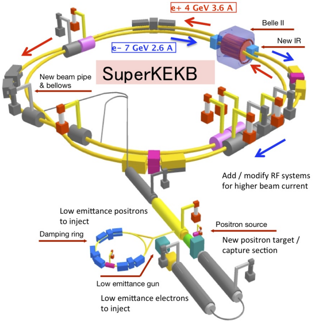


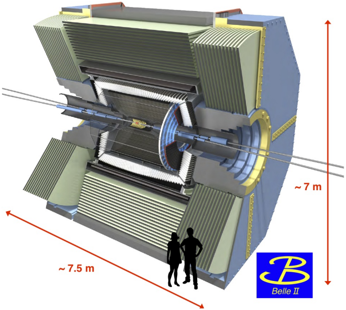
- KOTO @ J-PARC: $K_L^0 \to \pi^0 a$
  - Current status: taking data



- KLEVER @ CERN SPS: $K_L^0 \to \pi^0 a$
  - Current status: proposed (early stages) [Moulson '16]

- Belle(-II): $B^{\pm} \to K^{\pm}\nu\bar{\nu}$ and other B physics
  - Current status: calibrating



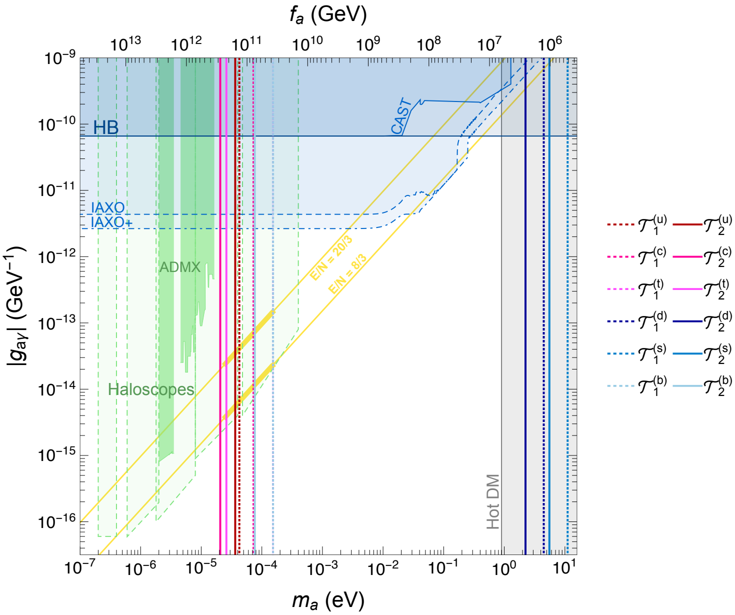


- What about D decays?
  - BESIII @ IHEP

Limits

Decay	Branching ratio	Experiment	$\tilde{c}_{P\to P'}$	$2f_a/\mathrm{GeV}$
$K^+ \to \pi^+ a$	$< 0.73 \times 10^{-10}$	E949 + E787	3.51×10^{-11}	$> 6.9 \times 10^{11} \lvert V_{21}^d \rvert$
	$< 0.01 \times 10^{-10}*$	NA62 (future)		$> 5.9 \times 10^{12} \lvert V_{21}^d \rvert$
	$< 1.2 \times 10^{-10}$	E949 + E787		
	$< 0.59 \times 10^{-10}$	E787		
$K_L^0 \to \pi^0 a$	$< 5 \times 10^{-8}$	KOTO	3.67×10^{-11}	$> 2.7 \times 10^{10} \lvert V_{21}^d \rvert$
$(K_L^0 \to \pi^0 \nu\bar{\nu})$	$(< 2.6 \times 10^{-8})$	E391a		
$B^\pm \to \pi^\pm a$	$< 4.9 \times 10^{-5}$	CLEO	5.30×10^{-13}	$> 1.0 \times 10^{8} \lvert V_{31}^d \rvert$
$(B^\pm \to \pi^\pm \nu\bar{\nu})$	$(< 1.0 \times 10^{-4})$	BaBar		
	$(< 1.4 \times 10^{-4})$	Belle		
$B^\pm \to K^\pm a$	$< 4.9 \times 10^{-5}$	CLEO	7.26×10^{-13}	$> 1.2 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^\pm \to K^\pm \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	BaBar		
	$(< 1.9 \times 10^{-5})$	Belle		
	$(< 1.5 \times 10^{-6})*$	Belle-II (future)		
$B^0 \to \pi^0 a$			4.92×10^{-13}	
$(B^0 \to \pi^0 \nu\bar{\nu})$	$(< 0.9 \times 10^{-5})$	Belle		$\gtrsim 2.3 \times 10^{8} \lvert V_{31}^d \rvert$
$B^0 \to K^0_{(S)} a$	$< 5.3 \times 10^{-5}$	CLEO	6.74×10^{-13}	$> 1.1 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^0 \to K^0 \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	Belle		
$D^\pm \to \pi^\pm a$	< 1		1.11×10^{-13}	$> 3.3 \times 10^{5} \lvert V_{21}^u \rvert$
$D^0 \to \pi^0 a$	< 1		4.33×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$D_s^\pm \to K^\pm a$	< 1		4.38×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$B_s^0 \to \overline{K}^0 a$	< 1		3.64×10^{-13}	$> 6.0 \times 10^{5} \lvert V_{31}^d \rvert$

Bounds in the $U(1)_{QF}$ model



Let us rotate away the anomaly term by

$$q \to e^{i\frac{\beta_q}{2}\frac{a}{f_a}\gamma_5} q, \qquad \beta_q = \frac{m_*}{m_q},$$

where $q = u, d, s$ and $m_*^{-1} = m_u^{-1} + m_d^{-1} + m_s^{-1}$. The axion-quark Lagrangian transforms as

$$\mathcal{L}_\partial \to \mathcal{L}'_\partial \supset -\frac{\partial_\mu a}{2f_a}\left[\sum_{q=u,d,s} c_q \bar{q}\gamma^\mu\gamma_5 q + c_{sd}\bar{s}\gamma^\mu\gamma_5 d + c_{sd}^* \bar{d}\gamma^\mu\gamma_5 s\right],$$

where

$c_u = A_{11}^u + \beta_u/2,$
$c_d = A_{11}^d + \beta_d/2,$
$c_s = A_{22}^d + \beta_s/2,$
$c_{sd} = A_{21}^d.$

We can write this as kinetic mixing between axions and mesons:

$$\mathcal{L}_{aP}^{\text{eff}} = -\sum_P c_P \frac{f_P}{2f_a} \partial_\mu a \partial^\mu P,$$

with

$$c_{\pi^0} = c_u - c_d, \qquad c_\eta = c_u + c_d - 2c_s$$
$$c_{\eta'} = c_u + c_d + c_s, \qquad c_{K^0} = c_{sd} = c_{\bar{K}^0}^*$$

Diagonalising the kinetic mixing,

$$a \to \frac{a}{\sqrt{1 - \sum_P \eta_P^2}}, \qquad P \to P + \frac{\eta_P a}{\sqrt{1 - \sum_P \eta_P^2}}$$

where

$$\eta_P \equiv \frac{c_P f_P}{2f_a}$$

Meson mass splitting

$$(\Delta m_P)_{\text{axion}} \simeq |\eta_P|^2 m_P = |c_P|^2 \frac{f_{P^0}^2}{(2f_a)^2} m_P.$$

System	$(\Delta m_P)_{\text{exp}}/\text{MeV}$	$2f_a/\text{GeV}$
$K^0 - \overline{K}^0$	$(3.484 \pm 0.006) \times 10^{-12}$	$\gtrsim 2 \times 10^6 \lvert c_{K^0}\rvert$
$D^0 - \overline{D}^0$	$(6.25\,^{+2.70}_{-2.90}) \times 10^{-12}$	$\gtrsim 4 \times 10^6 \lvert c_{D^0}\rvert$
$B^0 - \overline{B}^0$	$(3.333 \pm 0.013) \times 10^{-10}$	$\gtrsim 8 \times 10^5 \lvert c_{B^0}\rvert$
$B_s^0 - \overline{B}_s^0$	$(1.1688 \pm 0.0014) \times 10^{-8}$	$\gtrsim 1 \times 10^5 \lvert c_{B_s^0}\rvert$

PDG [Patrignani et al '16]

Notes

- Assume central SM value
- Uncertainty dominated by theory; require $(\Delta m_P)_{\text{axion}} \lesssim (\Delta m_P)_{\text{exp}}$
- Possible improvements to $(\Delta m_K)_{\text{th}}$ from lattice soon [Bai, Christ, Sachrajda '18]

Lepton decays proceed similarly to mesons. Define a total coupling

$$\left|C^e_{\ell_1\ell_2}\right|^2 = \left|V^e_{\ell_1\ell_2}\right|^2 + \left|A^e_{\ell_1\ell_2}\right|^2$$

Two-body decay branching ratio

$$\mathrm{Br}(\ell_1 \to \ell_2 a) = \frac{1}{16\pi\,\Gamma(\ell_1)} \frac{\left|C^e_{\ell_1\ell_2}\right|^2}{(2f_a)^2} m^3_{\ell_1} \left(1 - \frac{m^2_{\ell_2}}{m^2_{\ell_1}}\right)^3$$

We may also probe the angular distribution. For muons,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} \simeq \frac{|C^e_{21}|^2}{32\pi}\frac{m_\mu^3}{(2f_a)^2}(1 - AP_\mu \cos\theta)$$

where

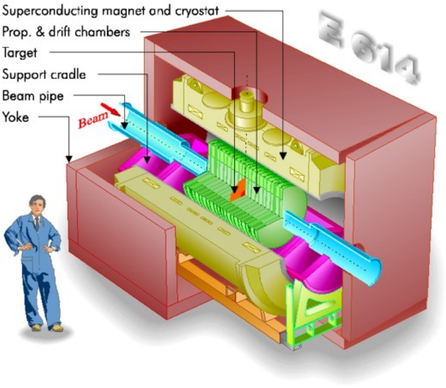
$$A = -\frac{2\mathrm{Re}[A^e_{21}(V^e_{21})^*]}{|C^e_{21}|^2}$$

Notes

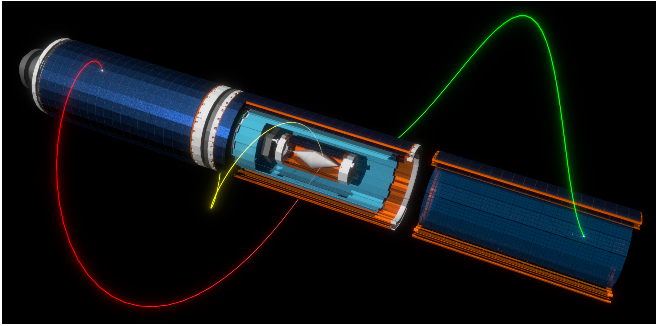
- Standard Model weak interactions are 'V-A' $\Leftrightarrow A = -1$
- Isotropic decays ($A = 0$) for $A^e_{21} = 0$ or $V^e_{21} = 0$.
- Strongest signal for 'V+A' (RH) interactions

- Jodidio *et al* @ TRIUMF [Jodidio *et al* '86]
  - Stopped μ^+ on metal foil
  - Assume isotropic decays ($A = 0$)

- TWIST @ TRIUMF [Bayes *et al* '14]
  - Sensitive to anisotropies
  - Limits for $A = 0$ not as good as TRIUMF



- Mu3e @ PSI
  - Stopped μ^+
  - Primary channel: $\mu^+ \to e^+ e^- e^+$
  - Also able to search for $\mu^+ \to e^+ X^0$ [Perrevoort (PhD thesis) '18]



Decay	Branching ratio	Experiment	$\tilde{c}_{\ell_1 \to \ell_2}$	$2f_a/\mathrm{GeV}$
$\mu^+ \to e^+ a$	$< 2.6 \times 10^{-6}$	$(A = 0)$ Jodidio *et al*	7.82×10^{-11}	$> 5.5 \times 10^9 \lvert V^e_{21}\rvert$
	$< 2.1 \times 10^{-5}$	$(A = 0)$ TWIST		$> 1.9 \times 10^9 \lvert C^e_{21}\rvert$
	$< 1.0 \times 10^{-5}$	$(A = 1)$ TWIST		$> 2.8 \times 10^9 \lvert C^e_{21}\rvert$
	$< 5.8 \times 10^{-5}$	$(A = -1)$ TWIST		$> 1.2 \times 10^9 \lvert C^e_{21}\rvert$
	$\lesssim 5 \times 10^{-9}*$	Mu3e (future)		$\gtrsim 1 \times 10^{11} \lvert C^e_{21}\rvert$
$\tau^+ \to e^+ a$	$< 1.5 \times 10^{-2}$	ARGUS	4.92×10^{-14}	$> 1.8 \times 10^6 \lvert C^e_{31}\rvert$
$\tau^+ \to \mu^+ a$	$< 2.6 \times 10^{-2}$	ARGUS	4.87×10^{-14}	$> 1.4 \times 10^6 \lvert C^e_{32}\rvert$

Decays like $\ell_1 \to \ell_2 a\gamma$, in the limit $m_{\ell_2} = m_a = 0$, may be expressed

$$\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\,\mathrm{d}y} = \frac{\alpha \left| C^e_{\ell_1\ell_2} \right|^2 m^3_{\ell_1}}{32\pi^2(2f_a)^2} f(x,y)$$

where

$$f(x,y) = \frac{(1-x)(2-y-xy)}{y^2(x+y-1)}, \quad x = \frac{2E_{\ell_2}}{m_{\ell_1}}, \quad y = \frac{2E_\gamma}{m_{\ell_1}}$$

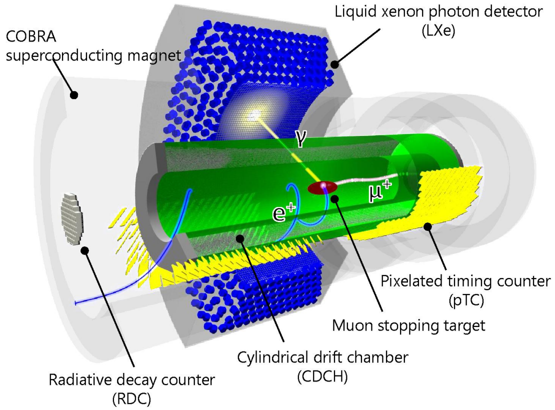
Kinematics and energy conservation fix

$$x, y \leq 1,\ x+y \geq 1,\ \cos\theta_{2\gamma} = 1 + \frac{2(1-x-y)}{xy}$$

Must consider

- IR divergences
- Experimental cuts (e.g. $E_\gamma > 40$ MeV in MEG)

- MEG(-II) @ PSI
  - Searching for $\mu \to e\gamma$ in stopped μ^+
  - Status: MEG completed, MEG-II under construction
  - Reach: TBD



Decay	Branching ratio	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	MEG
	$\lesssim 6 \times 10^{-14}*$	MEG-II (future)
$\tau^- \to e^-\gamma$	$< 3.3 \times 10^{-8}$	BaBar
$\tau^- \to \mu^-\gamma$	$< 4.4 \times 10^{-8}$	BaBar

Best limit on $\mu \to ef\gamma$ (for some scalar f)

- Crystal Box experiment [Bolton et al '88]
  - $\mathrm{Br}(\mu \to ef\gamma) < 1.1 \times 10^{-9}$
  - No assumptions on decay isotropy
- MEG-II should be more sensitive (full study needed)

Flavoured axion can mediate $\mu \to 3e$ through the $\mu e a$ vertex (t- and s-channel). To $\mathcal{O}(m_e^2)$, the branching ratio is

$$\begin{aligned}\mathrm{Br}(\mu^+ \to e^+ e^- e^+) &\approx \frac{m_e^2 m_\mu^3}{16\pi^3 \Gamma(\mu)} \frac{|A_{11}^e|^2 |C_{21}^e|^2}{(2f_a)^4} \left(\log \frac{m_\mu^2}{m_e^2} - \frac{15}{4} \right), \\ &\approx 1.43 \times 10^{-41} |A_{11}^e|^2 |C_{21}^e|^2 \left(\frac{10^{12}\ \mathrm{GeV}}{(2f_a)} \right)^4\end{aligned}$$

- Experiment: Mu3e @ PSI
  - Status: under construction, taking data in 2019
  - Reach: $\mathrm{Br} < \mathcal{O}(10^{-16})$
  - 4 OoM improvement over SINDRUM (1987)
  - $f_a \gtrsim 10^6$ GeV

The same $\mu e a$ vertex can mediate $\mu - e$ conversion in nuclei

$$R_{\mu e}^{(A,Z)} \equiv \frac{\Gamma(\mu^- \to e^-(A,Z))}{\Gamma_{\mu^-\mathrm{cap}}^{(A,Z)}}$$

$$\sim \frac{m_\mu^5}{(q^2 - m_a^2)^2} \frac{(\alpha Z)^3}{\pi^2 \, \Gamma_{\mu^-\mathrm{cap}}^{(A,Z)}} \frac{m_\mu^2 m_N^2}{(2f_a)^4} |C_{21}^e|^2 |S_N^{(A,Z)} C_{aN}|^2$$

- Spin-dependent process [see Cirigliano '17]
  - not seen: $\mathcal{O}(1)$ form factors
- Relevant couplings: C_{21}^e and $g_{aN} = C_{aN} m_N/(2f_a)$
  - C_{aN} is model-dependent, depends on diagonal charges
- Experiments
  - SINDRUM-II: current best limit $R_{\mu e}^{\mathrm{Au}} < 7 \times 10^{-13}$
  - Mu2e @ Fermilab and COMET @ J-PARC: under construction
  - Measure $R_{\mu e}^{\mathrm{Al}}$; both expected to reach 4 OoM improvement

Theory

- Generation-dependent $U(1)_{PQ} \Leftrightarrow$ flavoured axion.
- We have explored such a $U(1)$ quark flavour symmetry, with maximal reduction in free Yukawa parameters.
- Only two structures are allowed: both are PQ symmetries.
- Axion couplings are all fixed by flavour data (up to f_a).

Phenomenology

- Rare meson decays (esp. $K^+ \to \pi^+ a$)
- Neutral meson mixing [ALPs]
- Muon decays ($\mu^+ \to e^+ a$)
- $\mu \to 3e$ and $\mu - e$ conversion [ALPs]

1. Astrophysical bounds: g_{ae} and g_{aN}
2. Nucleophobia in the minimal $U(1)_{QF}$ model
3. Quark sequestration, and strong suppression of $K \to \pi a$
4. MEG-II: full analysis

Thank you!