Recent developments in top physics at hadron colliders

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Based on many papers with:

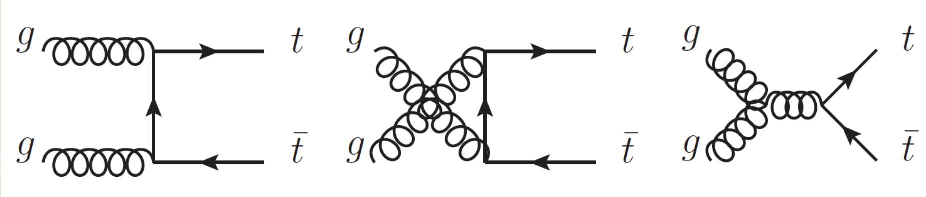
Barnreuther, Cacciari, Czakon, Fiedler, Mangano, Nason, Rojo, Sterman, Sung

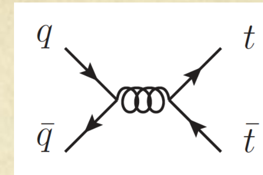
Content of the talk

- Few words about the historic developments
- Why is top production of interest (pheno)?
- How hard of a problem top production is?
  - Analytical properties
  - IR singularities
  - Gauge theory amplitudes
- Computing the NNLO: the methods.
- Precision applications at the LHC: what do we learn about SM and bSM?
- Outlook: the future of precision phenomenology.

Introduction to top production

In this talk I'll consider the process of top-pair production at hadron colliders



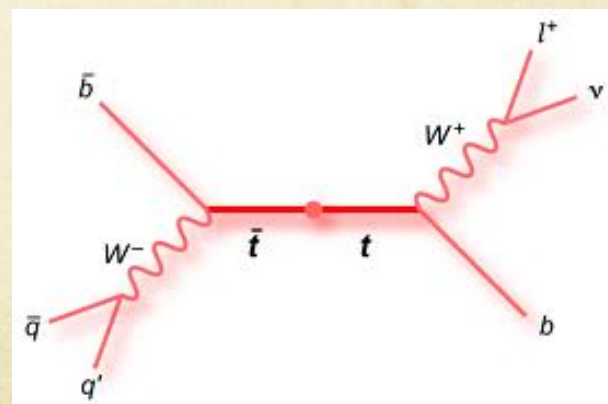


- The contributing partonic channels, and their relative contribution at LHC/Tevatron:

	TeVatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
gg	15.4%	84.8%	86.2%	90.2%
$qg + \bar{q}g$	-1.7%	-1.6%	-1.1%	0.5%
qq	86.3%	16.8%	14.9%	9.3%

- Top quarks decay very fast, so we never observe them directly. <u>They do not form bound states</u>.

- Will ignore their decay in this talk, and will consider them as stable particles (as if they are reconstructed in each event from their decay products – not true in reality).



In this talk I'll focus exclusively on the total inclusive x-section:

NOTE: differential distributions are well understood at NLO.
The total x-section is the first step into NNLO.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta \, \Phi_{ij}(\beta, \mu_F^2) \, \hat\sigma_{ij}(\beta, m^2, \mu_F^2, \mu_R^2)$$

Partonic fluxes
(derived from PDF's)

$$\Phi_{ij}(\beta, \mu_F^2) = \frac{2\beta}{1-\beta^2} \, \mathcal{L}_{ij}\left(\frac{1-\beta_{\rm max}^2}{1-\beta^2}, \mu_F^2\right)$$

$$\mathcal{L}_{ij}(x, \mu_F^2) = x \, (f_i \otimes f_j)\,(x, \mu_F^2)$$

Partonic x-section
(perturbative)

$$\hat\sigma_{ij}\,(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma_{ij}^{(0)} + \alpha_S \sigma_{ij}^{(1)} + \alpha_S^2 \sigma_{ij}^{(2)} + \mathcal{O}(\alpha_S^3)\right)$$

The partonic x-section depends on a single variable $\beta = \sqrt{1-\rho}$, with $\rho \equiv 4m^2/s$,

$0 < \rho \leq 1$

- ✓ Point β = 0 (absolute threshold)
- ✓ Point β = 1 (high energy limit, i.e. m=0)

Historic prospective

- ✓ Early NLO QCD results (inclusive, semi-inclusive)

  Nason, Dawson, Ellis '88
  Beenakker et al '89

- ✓ Nowadays: *the industry* of the NLO revolution, thanks to advances in NLO technology

  Bern, Dixon, Dunbar, Kosower `94
  Britto, Cachazo, Feng `04
  Ossola, Papadopoulos, Pittau `07
  Giele, Kunszt, Melnikov `08
  aMC@NLO

- ✓ Complete understanding at NLO:

  Bernreuther, Brandenburg, Si, Uwer
  Melnikov, Schulze
  Bevilacqua, Czakon, van Hameren, Papadopoulos, Wore
  Denner, Dittmaier, Kallweit, Pozzorini

- ✓ 1990's: the rise of the soft gluon resummation at NLL

  Kidonakis, Sterman '97
  Bonciani, Catani, Mangano, Nason `98

- ✓ NNLL resummation developed (and approximate NNLO approaches)

  Beneke, Falgari, Schwinn '09
  Czakon, Mitov, Sterman `09
  Beneke, Czakon, Falgari, Mitov, Schwinn `09
  Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11

- ✓ Electroweak effects at NLO known (small ~ 1.5%)

  Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth `93
  Hollik, Kollar `07
  Bernreuther, Fuecker, Si '05
  Kuhn, Scharf, Uwer '07

Main features of top-pair production

Top-pair production is completely understood within NLO/NNLL QCD

Main features:

- ✓ Large NLO QCD corrections
- ✓ Total theory uncertainty at (NLO+resummation)~10%
- ✓ Important for Higgs and bSM physics (M. Peskin: "*BSM Hides beneath Top*")
- ✓ Experimental improvements down to 5% (at LHC)
- ✓ Current LHC data agrees well with SM theory
- ✓ Tevatron data generally agrees too.

The notable exception: Forward-backward asymmetry from Tevatron.

Conclusion: "further scrutiny is needed"

Calculation of the total inclusive x-section tT @ NNLO during the last year

- Published qQ → tt +X

Bärnreuther, Czakon, Mitov `12

- Published all fermionic reactions (qq,qq',qQ')

Czakon, Mitov `12

- Published gq

Czakon, Mitov `12

- Published gg

Czakon, Fiedler, Mitov '13

Now the top pair total x-section is known numerically at NNLO in QCD

No (other) approximations of any kind

- First hadron collider calculation at NNLO with more than 2 colored partons.
- First NNLO hadron collider calculation with massive fermions.

- How to appreciate the complexity of the process?
- Let’s look at the NLO result which is analytically known

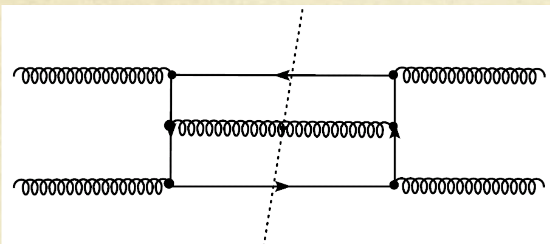
Based on: Czakon, Mitov arXiv:0811.4119

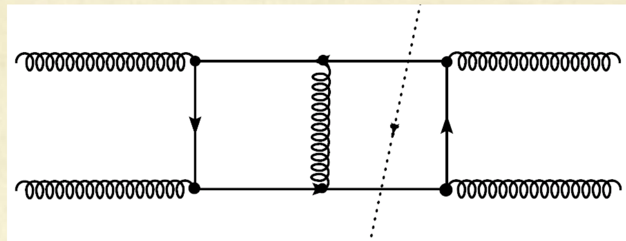
Recall, the NLO x-section first computed numerically

Nason, Dawson, Ellis '88
Beenakker, Kuijf, van Neerven, Smith, '89
Bernreuther, Brandenburg, Si, Uwer '04

Our strategy for the analytic computation:

- Treat Real and Virtual integrations on equal footing — Anastasiou, Melnikov '01
- Use IBP identities — Chetirkyn, Tkachov '81; Laporta '01





+ crossed

The NLO x-section has, approximately, the complexity of a 2-loop massive box

Our approach (it was a good approach):

- identify the possible physical singularities. There are 3 of them:
  - ✓ $m^2 \to 0$ (physical endpoint singularity),
  - ✓ $4m^2=s$ (physical endpoint singularity – partonic threshold),
  - ✓ $|m| \to \infty$ (unphysical singularity).
- change variables to map them to x=(-1,0,1)

$$\frac{m^2}{s} = \frac{x}{(1+x)^2} \qquad x = \frac{1-\sqrt{1-4\frac{m^2}{s}}}{1+\sqrt{1-4\frac{m^2}{s}}}$$

- one expects HPL's only.

- ✓ The whole x-section is mapped into 37 master integrals (real+virtual),
- ✓ We observe unexpected thing:
  - ➢ Few of the most complicated integrals (cross-box like) have additional singularities ("pseudothresholds")
- ✓ Their presence is expected in scattering amplitudes; but we have here a physical cross-section.
- ✓ We see them as additional singularities in the differential equations of the master integrals in the following points.

$s = m^2$; $s = -m^2$; $s = -4m^2$; $s = -16m^2$
(in addition to $s = 4m^2$ and $m^2=0$).

- ✓ They are outside the physical region, so no numerical problems,
- ✓ The problem is technical: no pure HPL solutions.

✓ The results for the qq and gq reactions in terms of simple polylogs

✓ The gg reaction involves 4 special functions

$$F_1(x) = -\int_x^1 dz \frac{(2z+1)(H(-1,0,z)+H(0,-1,z)-H(0,0,z))}{2(z^2+z+1)}$$

$$F_2(x) = -\int_x^1 dz \frac{(2z+3)(12H(-1,0,z)-6H(0,0,z)+\pi^2)}{4(z^2+3z+1)},$$

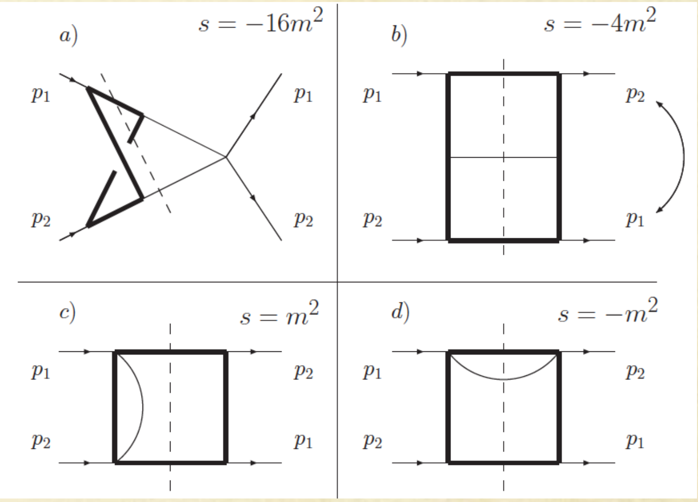
$$F_3(x) = +\int_x^1 dz \frac{5(z-1)(12H(-1,0,z)-6H(0,0,z)+\pi^2)}{8z\sqrt{z^2+6z+1}}.$$

$$F_4(x) = \int_\rho^1 d\tau\, I_4(\rho,\tau)$$

$$I_4(\rho,\tau) = \frac{45\rho}{32\pi\tau}\log\left(\frac{1-\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)\left(\frac{\left((\rho^2+1)K(\sqrt{-4\rho})-(\rho-1)E(\sqrt{-4\rho})\right)K\left(\frac{1}{\sqrt{4\tau+1}}\right)}{\sqrt{4\tau+1}} + \frac{\left((-4\rho^2+3\rho+1)E\left(\frac{1}{\sqrt{4\rho+1}}\right)+(3\rho^2-3\rho-2)K\left(\frac{1}{\sqrt{4\rho+1}}\right)\right)K(\sqrt{-4\tau})}{\sqrt{4\rho+1}}\right).$$

Elliptic functions of I and II kind

- The structure of the solution is such that it does not allow iterative solution.
- Clear example where it is important to know what the class of solutions is
- Reached beyond where the symbols are useful?
- I am unaware of other example of observable with such unphysical singularities.



Our conclusion: pursue a numerical approach for NNLO

Before the exact NNLO was computed, we knew:

- NNLO in threshold region and soft-gluon resummation at NNLL
- singularities of massive 2-loop gauge theory amplitudes

Soft-gluon resummation at hadron colliders
(and top production in particular)

What is soft-gluon resummation?

- The effect is mostly driven by kinematics:

Sterman ‘87
Catani, Trentadue ‘89

  - the system is in a corner of phase space where only soft gluons can be emitted
  - multiple emissions from semi-classical (eikonal) partons
  - Low scales -> large coupling.
  - Soft resummation is an alternative expansion not in “fixed coupling” but in “fixed Log”

- “Easy” for “standard” processes: Higgs, Drell-Yan, DIS, e^+e^-

Key: the number of hard colored partons < 4

- Harder for top production (there are color correlations for n>=4)

Non-trivial color algebra in this case.

- NLL resummation for top developed
  - For total inclusive
  - For differential

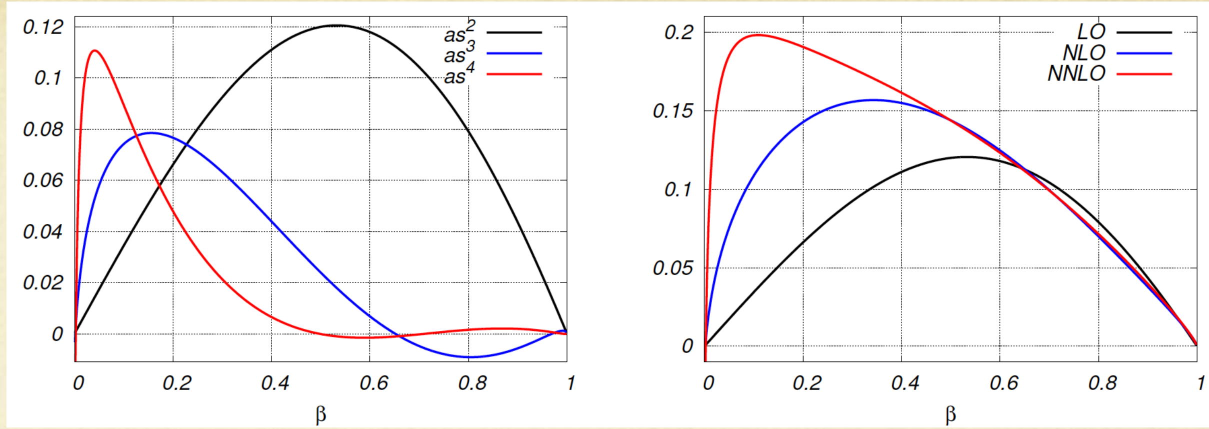
Bonciani, Catani, Mangano, Nason `98
Sterman, Kidonakis, Oderda `96-`98

“Patch” an observable in any kinematical region where usual perturbative expansion breaks down

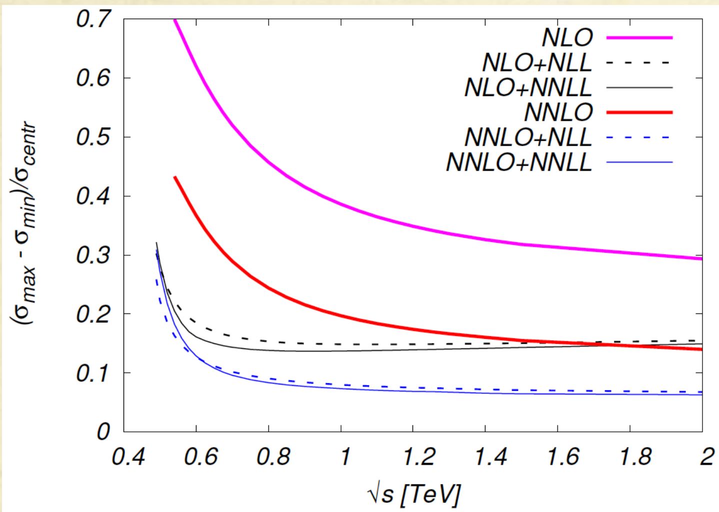
Soft-gluon resummation: an example

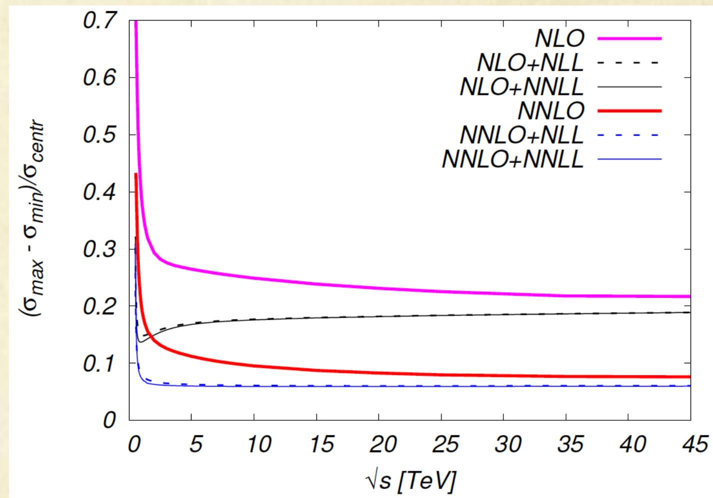
Partonic x-section's growth close to threshold (qq reaction):

The expansion there is not converging
Resummation needed



$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \ldots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \ldots\right)$$





Update of: Cacciari, Czakon, Mangano, Mitov, Nason '11

The resummed results are better close to threshold, as expected.

The top cross-section: NNLL resummation

Factorization of the partonic cross-section close to threshold:

Kidonakis, Sterman '97
Czakon, Mitov, Sterman '09

$$\omega_P\left(N,\hat{\eta},\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right) = J_1(N,\alpha_s(\mu^2))\dots J_k(N,M/\mu,m/\mu,\alpha_s(\mu^2)) \times \mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2},\frac{M^2}{m^2},\alpha_s(\mu^2)\right)\right] + \mathcal{O}(1/N)$$

N – the usual Mellin dual to the kinematical variable
that defines the threshold kinematics:

$$\sigma(N) = \int_0^1 dz z^{N-1}\sigma(z)$$

$z = Q^2/s$ ← Drell-Yan

$z = 4m^2/s$ ← t-tbar total X-section

$z = M_{t\bar{t}}^2/s$ ← t-tbar – pair invariant mass

J's – jet functions (different from the ones in amplitudes)

S,H – Soft/Hard functions. Also different.

The top cross-section: NNLL resummation

Here is the result for the Soft function:

$$
\begin{aligned}
\mathbf{S}\left(\frac{N^2\mu^2}{M^2},\beta_i\cdot\beta_j,\alpha_s(\mu^2)\right)\Bigg|_{\mu=M} &= \overline{\mathcal{P}}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S^{\dagger}\left(\beta_i\cdot\beta_j,\alpha_s\left(\mu'^2\right)\right)\right\}\\
&\quad\times\mathbf{S}\left(1,\beta_i\cdot\beta_j,\alpha_s\left(M^2/\bar{N}^2\right)\right)\\
&\quad\times\mathcal{P}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j,\alpha_s\left(\mu'^2\right)\right)\right\}\\
&= \overline{\mathcal{P}}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S^{\dagger}\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\}\\
&\quad\times\mathbf{S}\left(1,\beta_i\cdot\beta_j,\alpha_s\left(M^2/N^2\right)\right)\\
&\quad\times\mathcal{P}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\}
\end{aligned}
$$

Note: the Soft function satisfies RGE with the same anomalous dimension matrix as the Soft function of the underlying amplitude!

Therefore: knowing the singularities of an amplitude, allows resummation of soft logs in observables!

Singularities of Massive Gauge Theory Amplitudes

Amplitudes: the basics

- Gauge theory amplitudes: UV renormalized, S-matrix elements
- The amplitudes are not observables:
  - UV renormalized gauge amplitudes are not finite due to IR singularities.
  - Assume they are regulated dimensionally $d=4-2\varepsilon$

Some prior general results

✓ Explicit expression for the IR poles of any one-loop amplitude derived

Catani, Dittmaier, Trocsanyi '00

✓ The small mass limit is proportional to the massless amplitude

Mitov, Moch '06
Becher, Melnikov '07

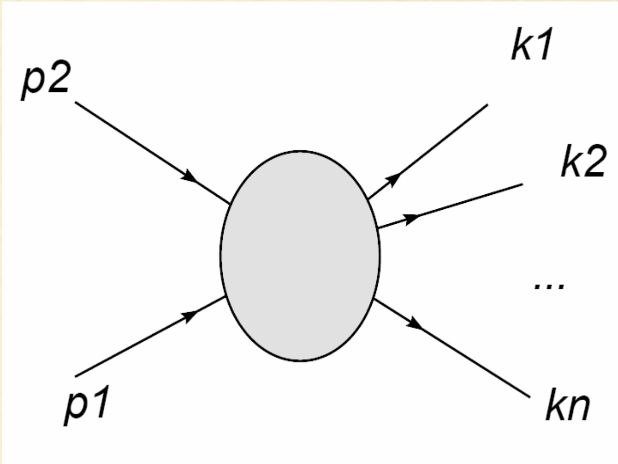
Note: predicts not just the poles but the finite parts too (for $m \to 0$)!

Factorization: "divide and conquer"

Structure of amplitudes becomes transparent thanks to factorization th.

$$M_I(\varepsilon, \mu_R, s_{ij}, m_i) = J(\varepsilon, \mu_R, \mu_F, m_i) \cdot S_{IJ}(\varepsilon, \mu_R, \mu_F, s_{ij}, m_i) \cdot H_J(\varepsilon, \mu_R, \mu_F, s_{ij}, m_i)$$

Note: applicable to both massive and massless cases



I,J – color indexes.

J(…) – "jet" function. Absorbs all the collinear enhancement.

S(…) – "soft" function. All soft non-collinear contributions.

H(…) – "hard" function. Insensitive to IR.

Factorization: the Jet function

$$M_I(Q,m,\epsilon) = J(m,\epsilon)\cdot S_{IJ}(Q,m,\epsilon)\cdot H_J(Q,m)$$

For an amplitude with n-external legs, J(…) is of the form:

$$J(m,\epsilon) = \prod_{i=1}^{n} J_i(m,\epsilon)$$

i.e. we associate a jet factor to each external leg.

Some obvious properties:
- Color singlets,
- Process independent; i.e. do not depend on the hard scale Q.

J_i not unique (only up to sub-leading soft terms).

A natural scheme: J_i = square root of the space-like QCD formfactor.

Sterman and Tejeda-Yeomans ‘02

Scheme works in both the massless and the massive cases.

The massive form-factor’s exponentiation known through 2 loops

Mitov, Moch ‘06

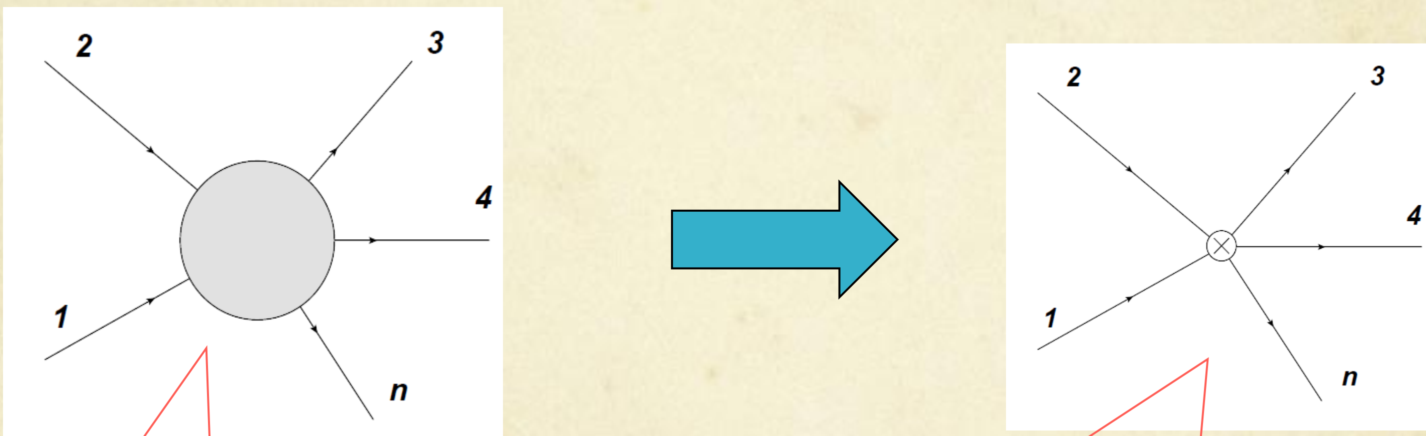
Factorization: the Soft function

$$M_I(Q,m,\epsilon) = J(m,\epsilon)\cdot S_{IJ}(Q,m,\epsilon)\cdot H_J(Q,m)$$

Soft function is the most non-trivial element
(recall: it contains only soft poles).

But we know that the soft limit is reproduced by the eikonal approximation.

➔ Extract S(…) from the eikonalized amplitude:



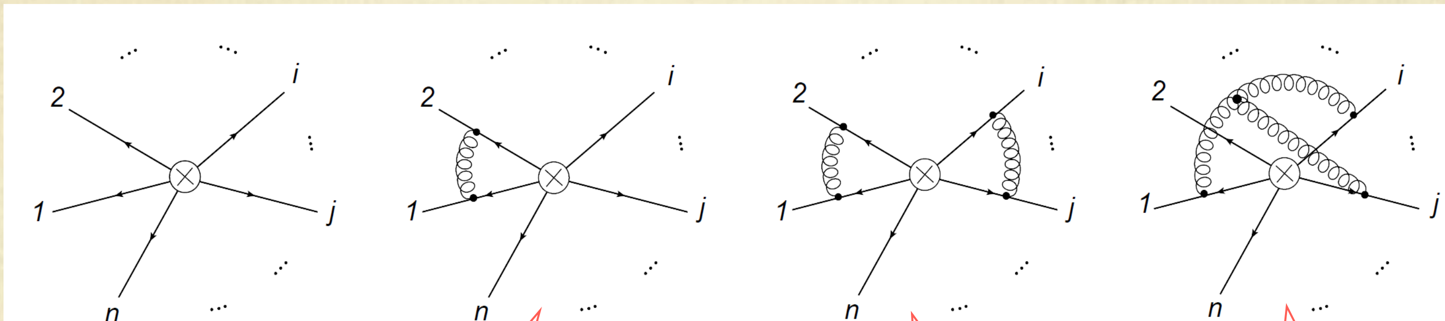
The LO amplitude M(…)

The eikonal version of the amplitude.
(the blob is replaced by an effective n-point vertex)

Factorization: the Soft function

Calculation of the eikonal amplitude:

consider all soft exchanges between the external (hard) partons



The fixed order expansion of the soft function takes the form:

$$
\begin{aligned}
S^{(1)}_{IJ}(\varepsilon, s_{ij}, m_i) &= \frac{1}{\varepsilon}\Gamma^{(1)}_{IJ}(s_{ij}, m_i) + O(\varepsilon^0)\,, \\
S^{(2)}_{IJ}(\varepsilon, s_{ij}, m_i) &= -\frac{\beta_0}{4\varepsilon^2}\Gamma^{(1)}_{IJ}(s_{ij}, m_i) + \frac{1}{2}\left(S^{(1)}_{IJ}(\varepsilon, s_{ij}, m_i)\right)^2 + \frac{1}{\varepsilon}\Gamma^{(2)}_{IJ}(s_{ij}, m_i) + O(\varepsilon^0)\,.
\end{aligned}
$$

… as follows from the usual RG equation:

$$
\left(\mu\frac{\partial}{\partial\mu} + \beta(g,\varepsilon)\frac{\partial}{\partial g}\right) S_{IJ}(\varepsilon, s_{ij}, m_i) = -\Gamma_{IK}(\varepsilon, s_{ij}, m_i)\; S_{KJ}(\varepsilon, s_{ij}, m_i)
$$

➔ All information about S(…) is contained in the anomal's dimension matrix Γ_{IJ}

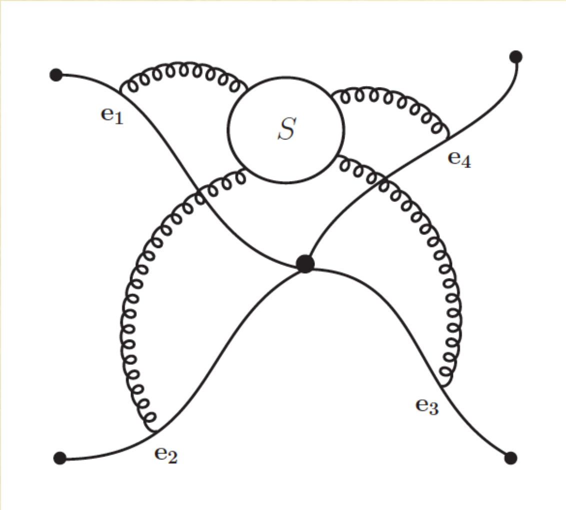
Factorization: the Soft function

How to define and compute these diagrams?

These diagrams are known as “webs”. Developed initially for color-singlet vertices.

Gatheral ’83
Frenkel and J. C. Taylor ’84
Sterman ‘81

General case now formulated, too



Mitov, Sterman, Sung ‘10
Gardi, Laenen, Stavenga, White ‘10

✓ The two-loop case is completely solved in QCD (massless and massive cases).

✓ Partial results at three loops.

Gardi et al
Becher, Neubert

the Soft function at 1 loop

Here is the result for the anomalous dim. matrix at one loop

$$\mathbf{\Gamma}_S^{(1)} = \frac{1}{2}\sum_{(i\neq j)=1}^{n} T_i \cdot T_j \ln\left(-\frac{\mu^2}{\sigma_{ij}}\right) + \frac{1}{2}\sum_{(i\neq j)\in\mathcal{N}_m} T_i \cdot T_j \left[\ln\left(1+x_{ij}^2\right) + \frac{2x_{ij}^2}{1-x_{ij}^2}\ln(x_{ij})\right]$$

The massless case

O(m) corrections in the massive case

where:
- all masses are taken equal,
- written for space-like kinematics (everything is real).

$$\frac{m^2}{s_{ij}} = -\frac{x_{ij}}{(1-x_{ij})^2} \quad , \quad x_{ij} = \frac{\sqrt{1-\frac{4m^2}{s_{ij}}}-1}{\sqrt{1-\frac{4m^2}{s_{ij}}}+1}$$

$$s_{ij} = (p_i + p_j)^2 \text{ and } \sigma_{ij} = 2p_i \cdot p_j = s_{ij} - m_i^2 - m_j^2$$

The Soft function at 2 loops

The simplest approach is the following. Start with the Ansatz:

$$\mathbf{\Gamma}_S^{(2)} = \frac{1}{2} \sum_{(i \neq j)=1}^{n} T_i \cdot T_j \, \frac{K}{2} \, \ln\left(-\frac{\mu^2}{\sigma_{ij}}\right) + \frac{1}{2} \sum_{(i \neq j) \in \mathcal{N}_m} T_i \cdot T_j \, P_{ij}^{(2)} + 3E \text{ terms}$$

Reproduces the massless case

Parametrizes the O(m) corrections to the massless case

Then note: the function $P^{(2)}_{ij}$ depends on (i,j) only through s_{ij}

➔ $P^{(2)}_{ij} = P^{(2)}(s_{ij})$

This single function can be extracted from the known n=2 amplitude: the massive two-loop QCD formfactor.

Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi `04
Gluza, Mitov, Moch, Riemann '09

The Soft function at 2 loops

The complete result for the 2E reads: $P^{(2)} = \frac{K}{2} P^{(1)} + P^{(2),\mathrm{m}}$

$$P^{(2),\mathrm{m}}(x) = \frac{C_A}{(1-x^2)^2} \Big\{ -\frac{(1+x^2)^2}{2}\mathrm{Li}_3(x^2) + \left(\frac{(1+x^2)^2}{2}\ln(x) - \frac{1-x^4}{2}\right)\mathrm{Li}_2(x^2) + \frac{x^2(1+x^2)}{3}\ln^3(x) + x^2(1-x^2)\ln^2(x) + \left(-(1-x^4)\ln\left(1-x^2\right) + x^2(1+x^2)\zeta_2\right)\ln(x) + x^2(1-x^2)\zeta_2 + 2x^2\zeta_3 \Big\},$$

This term breaks the simple relation $\boldsymbol{\Gamma}^{(2)}_{S_\mathrm{f}} = \frac{K}{2}\boldsymbol{\Gamma}^{(1)}_{S_\mathrm{f}}$ from the massless case!

Aybot, Dixon, Sterman '06

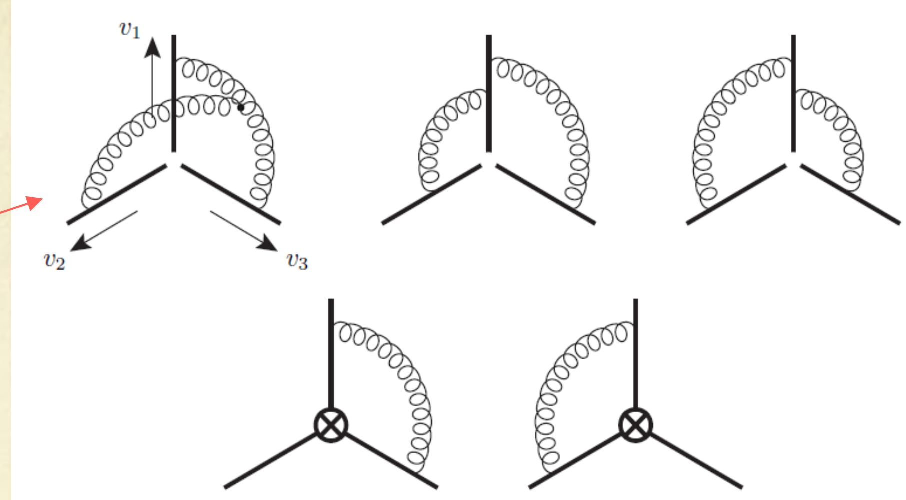
Above result derived by 3 different groups:

Kidonakis '09
Becher, Neubert '09
Czakon, Mitov, Sterman '09

Kidonakis derived the massive eikonal formfactor;
Becher, Neubert used old results of Korchemsky, Radushkin

The Soft function at 2 loops. The 3E diagrams.

The types of contributing diagrams:



The analytical result is very simple:

Ferroglia, Neubert, Pecjak, Yang ’09

$$F^{(3g)} \sim \sum_{ijk} \epsilon^{ijk} \ln^2(x_{ij})\, r(x_{ik})$$

where:

$$r(x) = -\frac{1+x^2}{1-x^2}\ln(x)$$

Recall:

it vanishes in the massless case, which makes the relation $\Gamma^{(2)}_{S_f} = \frac{K}{2}\Gamma^{(1)}_{S_f}$ possible.

Aybat, Dixon and Sterman ‘06

Massive gauge amplitudes: Summary

- The results I presented can be used to predict the poles of any massive 2-loop amplitude with:
  - *n* external colored particles (plus arbitrary number of colorless ones),
  - arbitrary values of the masses (usefull for SUSY).

- Results checked in the 2-loop amplitudes:

$$\langle M^{(2)} | M^{(0)} \rangle (q\bar{q} \to Q\overline{Q})$$
$$\langle M^{(2)} | M^{(0)} \rangle (gg \to Q\overline{Q})$$

- Needed in jet subtractions with massive particles at 2-loops

- Input for NNLL resummation

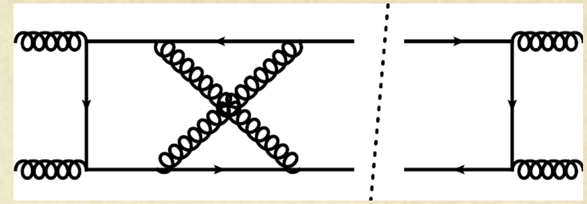
- Next frontier: 3-loop anomalous dimension matrix

- Application of webs to N=4 SUSY

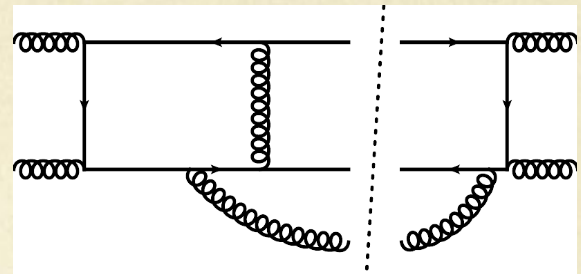
Calculation of the top-pair x-section at NNLO

What's needed for NNLO?

There are 3 principle contributions:

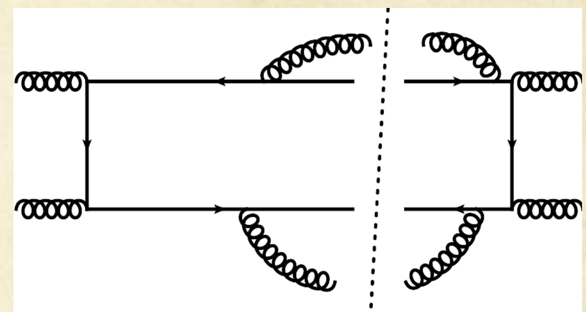


✓ 2-loop virtual corrections (V-V)



✓ 1-loop virtual with one extra parton (R-V)

✓ 2 extra emitted partons at tree level (R-R)



And 2 secondary contributions:

✓ Collinear subtraction for the initial state

Known, in principle. Done numerically.

✓ One-loop squared amplitudes (analytic)

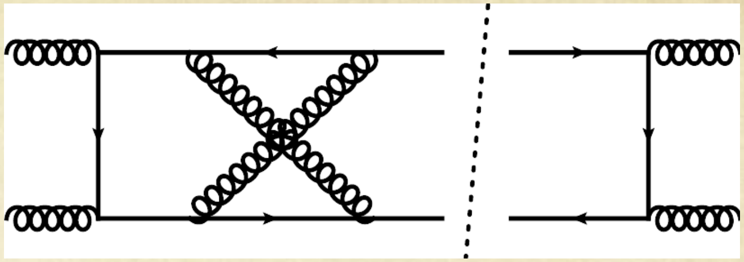
May be avoided?

Korner, Merebashvili, Rogal `07
Anastasiou, Mert-Aybot `08

Weinzierl `11

What's needed for NNLO? V-V

The two-loop amplitude gg → QQ:

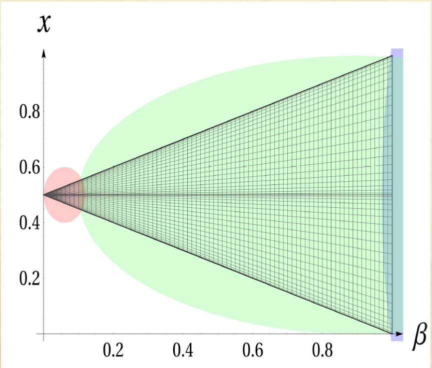


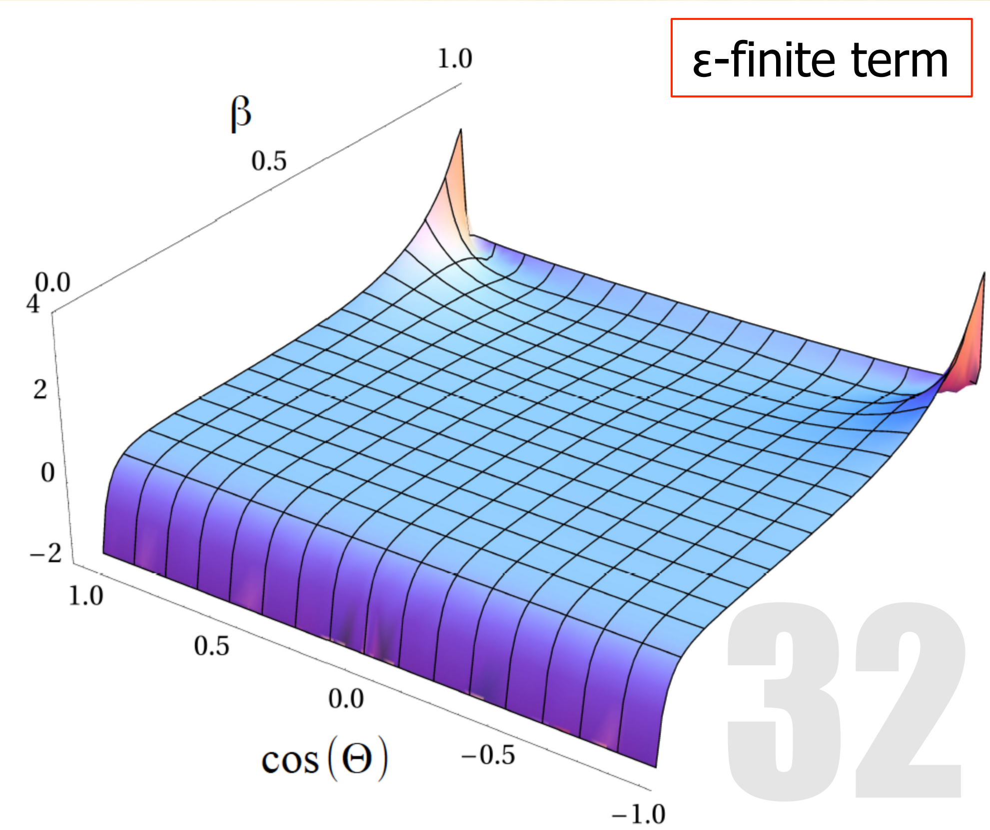
- ✓ Computed numerically
  Bärnreuther, Czakon, Fiedler '13
- ✓ (method similar to qq → QQ)
  Czakon `07
- ✓ Number of color structures known analytically
  Bonciani, Ferroglia, Gehrmann, von Manteuffel, Studerus
- ✓ High-energy limit and poles known analytically

System of 422 masters of 2 variables

$$x \equiv \frac{m^2 - \hat{t}}{\hat{s}} = \frac{1}{2}\left(1 - \beta\cos(\Theta)\right)$$

Integrated numerically





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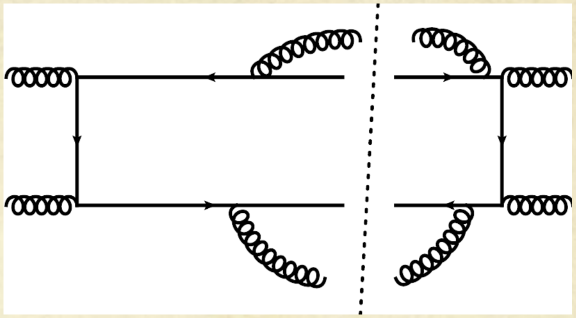
What’s needed for NNLO? R-R



✓ A wonderful result By M. Czakon

Czakon `10-11

✓ The method is general (also to other processes, differential kinematics, etc).

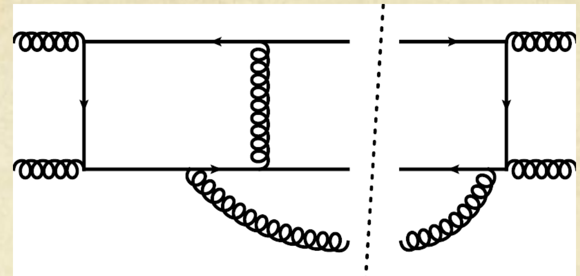
✓ Explicit contribution to the total cross-section given.

✓ Just been verified in an extremely non-trivial problem.

✓ Applied to other processes too (H+j)

Boughezal, Caola, Melnikov, Petriello, Schulze ‘13

What's needed for NNLO? R-V



✓ Counterterms all known (i.e. all singular limits)

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini '00
Bierenbaum, Czakon, Mitov '11

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

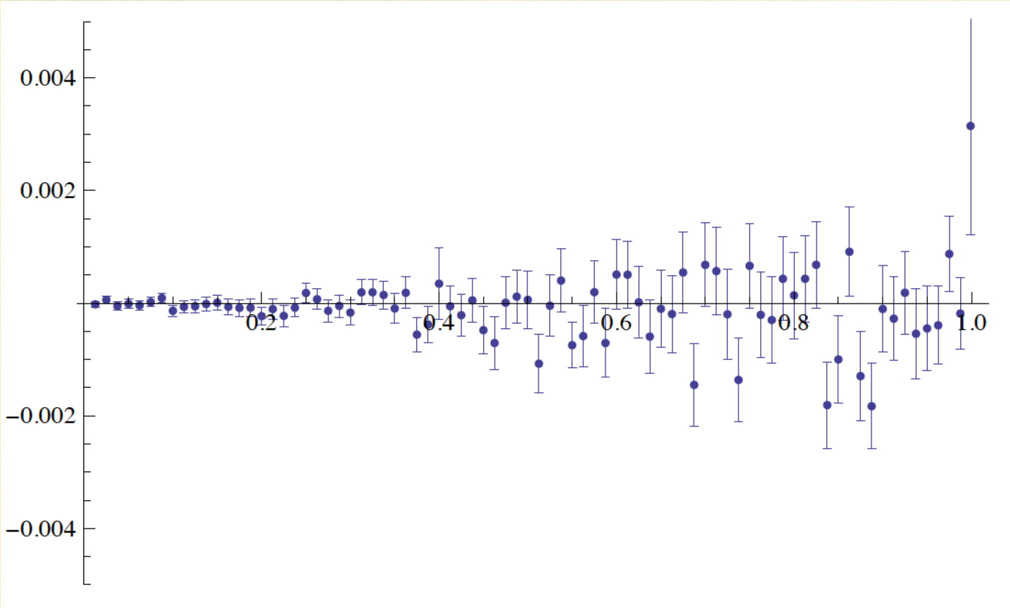
Extremely fast code!

A great help!

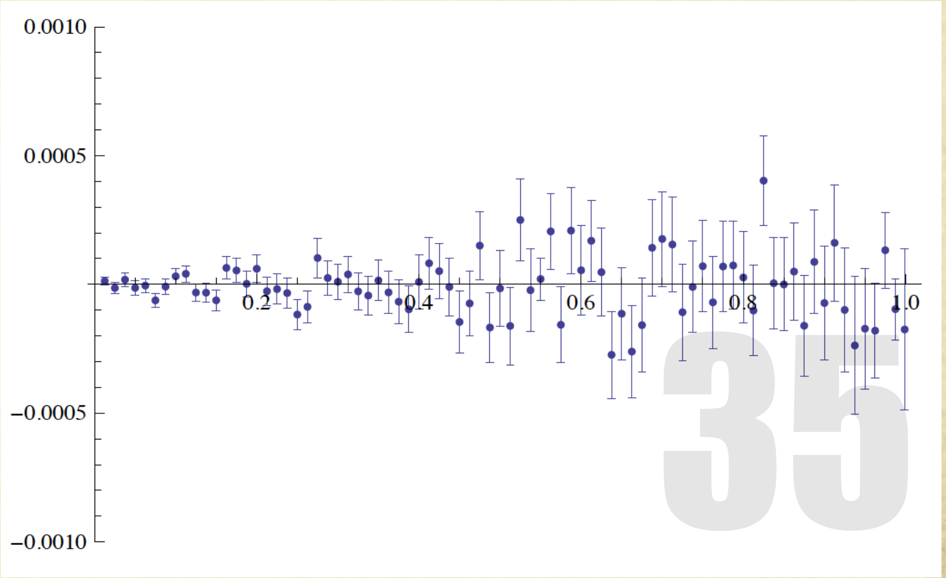
Many thanks!

A note on the calculation

✓ Will only show the cancellation of the deepest singularity 1/ε in gg-> tt:



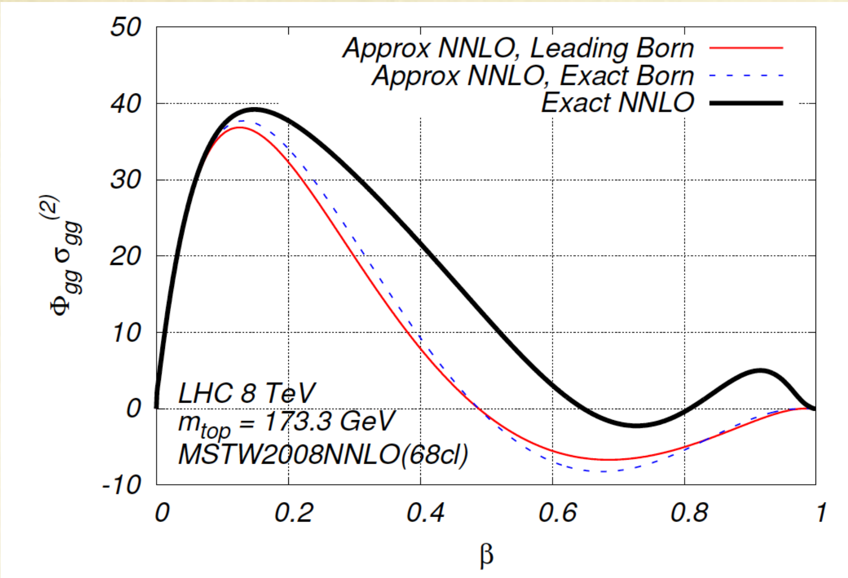
✓ And for $1/\varepsilon^2$ in gg-> tt:



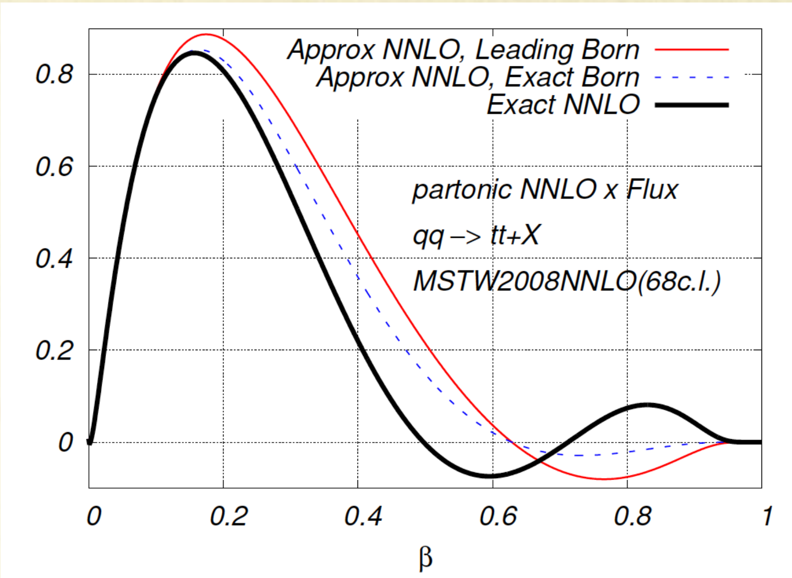
Parton level results

Partonic NNLO cross-sections, convoluted with LHC/Tevatron partonic fluxes

Czakon, Fiedler, Mitov '13



Bärnreuther, Czakon, Mitov `12



Note the agreement between the exact result and the threshold approximation
Derived from soft-gluon resummation + bound state effects

- The exact result is computed numerically, in 80 points on the interval 0<beta<1

Results @ parton level: gg -> ttbar +X

Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\,\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\,\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+\mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

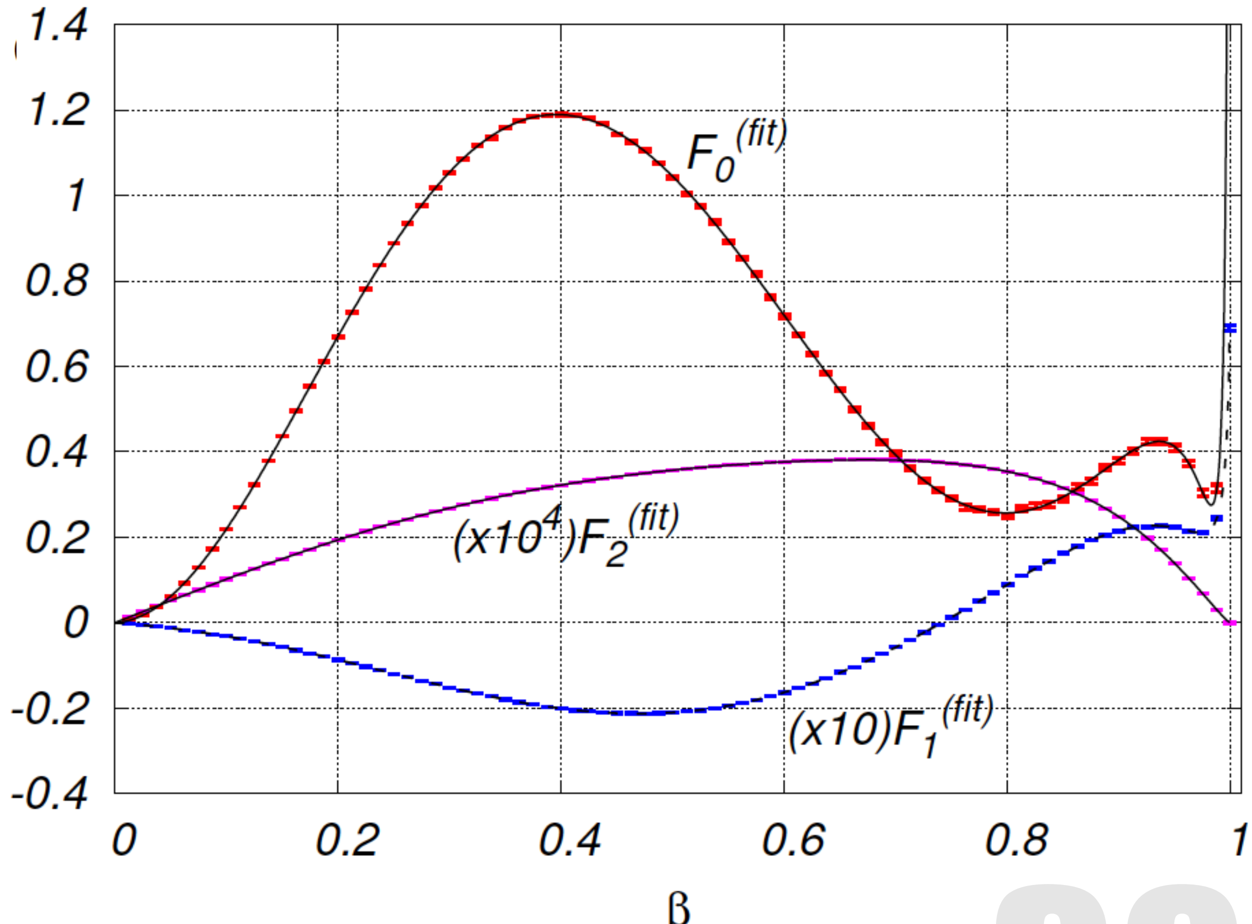
$$\sigma_{gg}^{(2)}(\beta)=F_0(\beta)+F_1(\beta)N_L+F_2(\beta)N_L^2$$

Numeric

$$F_i\equiv F_i^{(\beta)}+F_i^{(\rm fit)},\ i=0,1,2$$

The known threshold approximation

Beneke, Czakon, Falgari, Mitov, Schwinn `09



Czakon, Fiedler, Mitov '13

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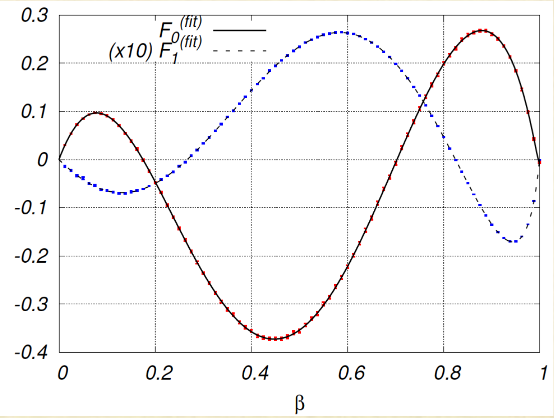
Results @ parton level:
The all-fermionic reactions

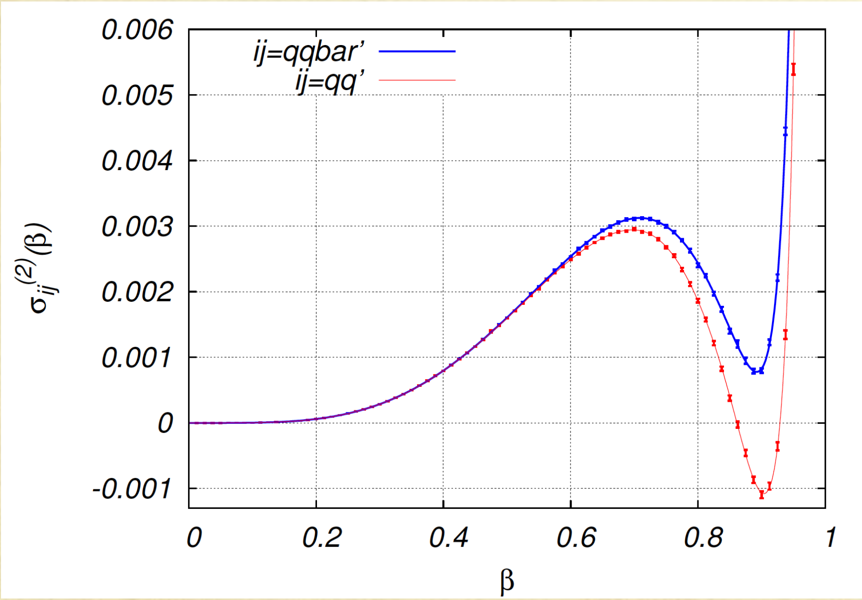
$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\rm NS}\,,$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}'\,,$$
$$qq' \to t\bar{t} + qq'\,,$$
$$qq \to t\bar{t} + qq\,.$$

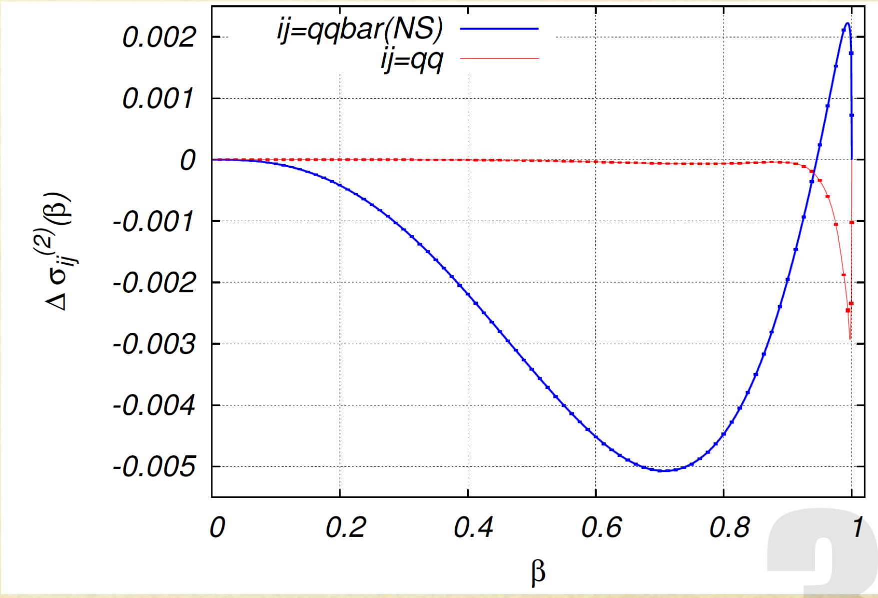


Czakon, Mitov '12

P. Bärnreuther et al arXiv:1204.5201

These partonic cross-sections are very small.
Compare to the ones involving qqbar!

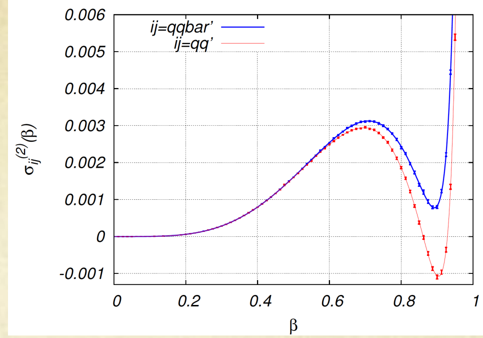


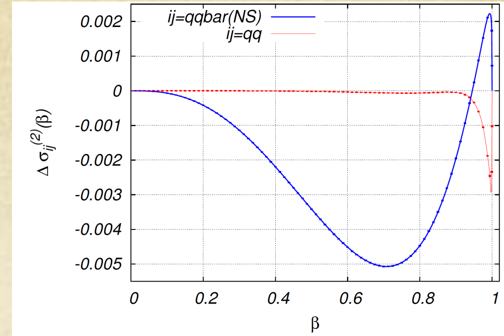


✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:
The all-fermionic reactions

$$\begin{aligned} q\bar{q} &\to t\bar{t} + q\bar{q}\big|_{\rm NS}\,, \\ q\bar{q}' &\to t\bar{t} + q\bar{q}'\,, \\ qq' &\to t\bar{t} + qq'\,, \\ qq &\to t\bar{t} + qq\,. \end{aligned}$$





The interesting feature: high-energy logarithmic rise:

$$\sigma^{(2)}_{f_1 f_2 \to t\bar{t} f_1 f_2}\Big|_{\rho\to 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho) \qquad \rho = \frac{4m_t^2}{s}$$

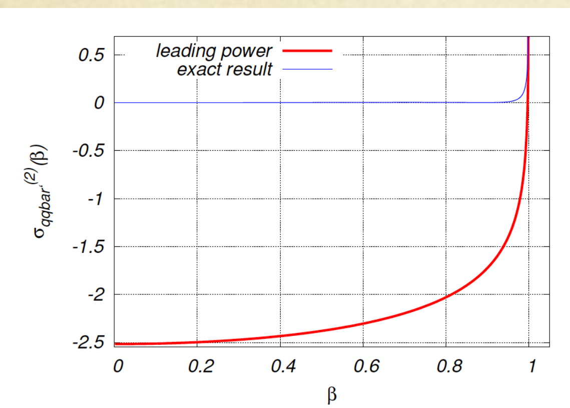
$$c_1 = -0.4768323995789214$$

Known analytically

Ball, Ellis `01

$$c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from} \quad \sigma^{(2)}_{q\bar{q}'} \\ -2.5186 & \text{from} \quad \sigma^{(2)}_{qq'} \end{cases}$$

- Direct extraction from the fits.
  5% uncertainty. Czakon, Mitov '12
- Agrees with independent prediction.
  50% uncertainty. Moch, Uwer, Vogt '12



High-energy expansion non-convergent.

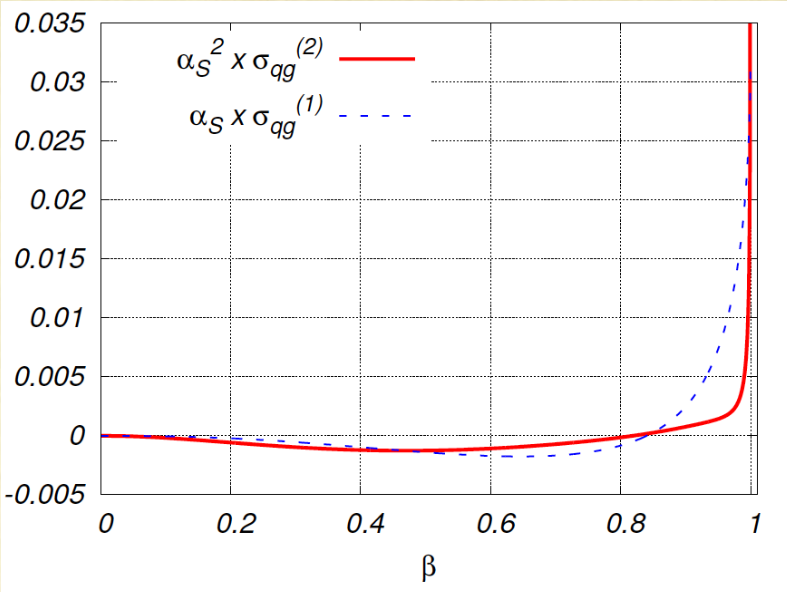
Applies only to the high-energy limit.

	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
$\sigma_{\rm all}$ [pb]	0.0003	0.0970	0.1504	0.7885
$\sigma_{\rm tot}$ [pb]	7.0056	154.779	220.761	852.177

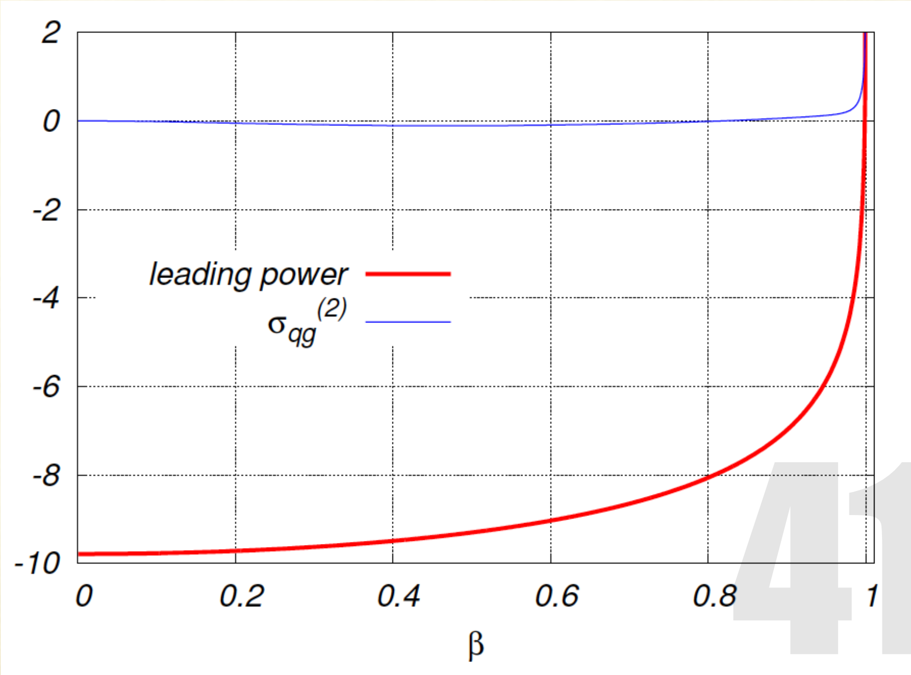
Czakon, Mitov '12

Results @ parton level:qg -> ttbar +X

Czakon, Mitov `12



- ✓ Correction about -1% (Tev and LHC).
- ✓ Notable decrease of scale dependence at LHC.
- ✓ NNLO large compared to NLO.



- ✓ High-energy log-limit correct

Ball, Ellis `01

- ✓ Agree for the constant with

Moch, Uwer, Vogt '12

- ✓ The limit itself plays no Pheno role

Checking the high-energy limit approximation

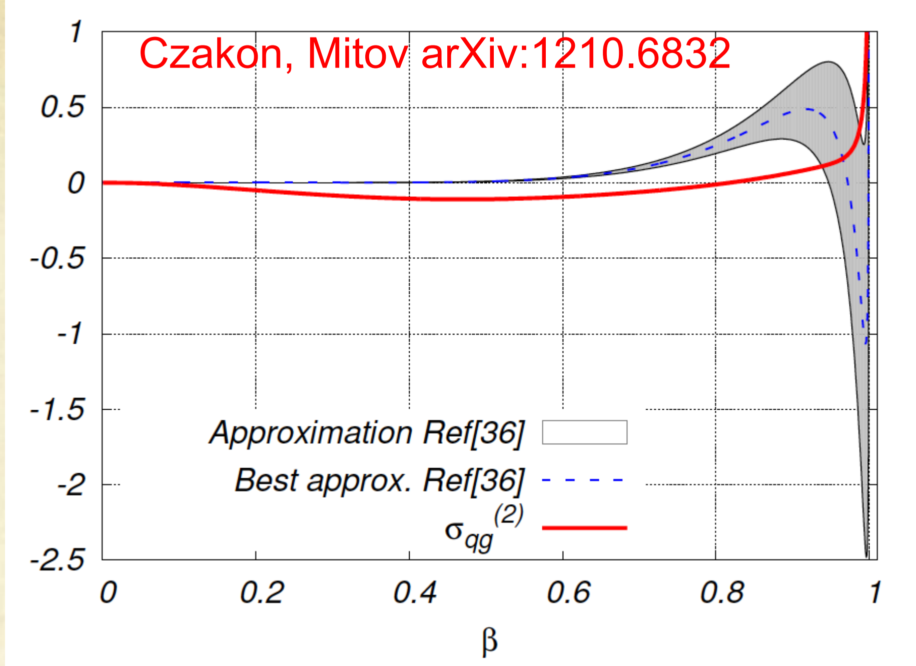
- ✓ It was suggested to use the high-energy limit of the X-section to predict it everywhere:

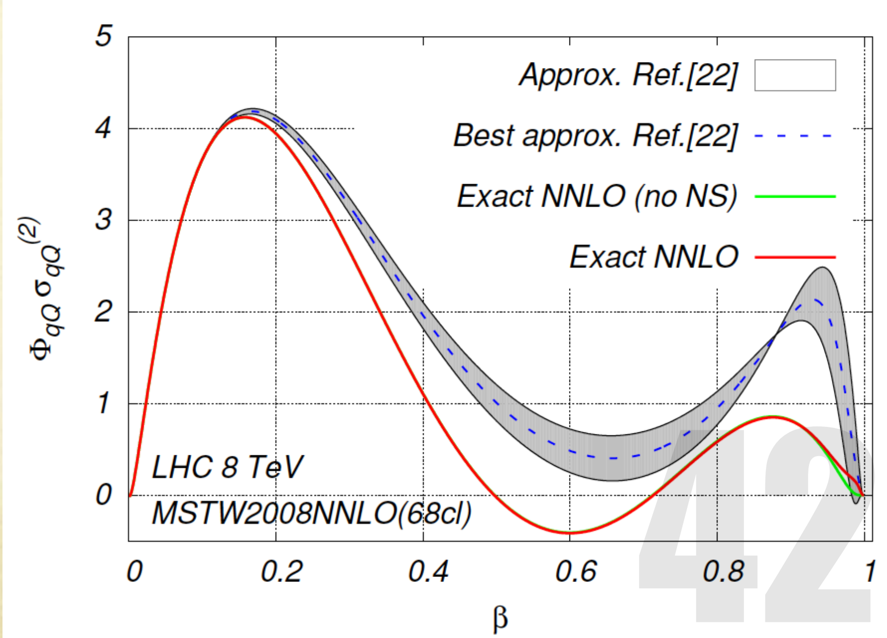
Moch, Uwer, Vogt ’12

- ✓ MUV approximation dramatically deviates from the exact gq NNLO result

- ✓ Leads to large difference for the x-section O(5%) from gq alone !

- ✓ Similar deviation for qq->tT+X (flux included)





Precision phenomenological applications

Prediction at NNLO+ resummation (NNLL)

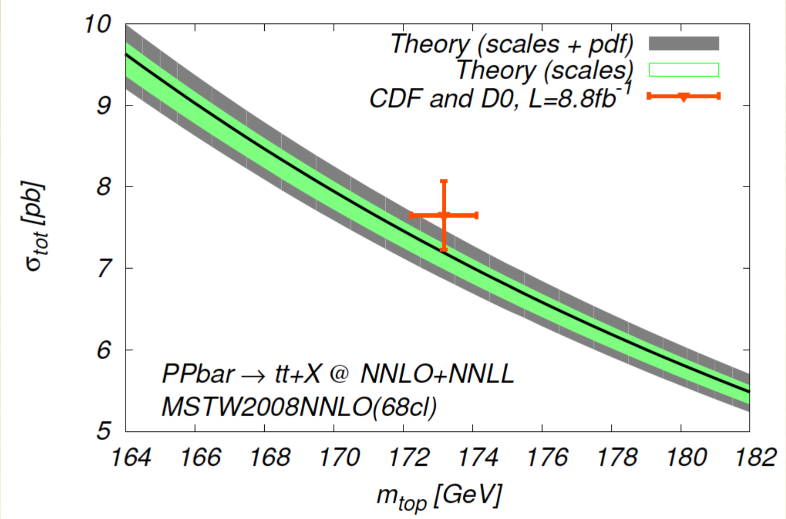
Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$+0.110(1.5\%)$ $-0.200(2.8\%)$	$+0.169(2.4\%)$ $-0.122(1.7\%)$
LHC 7 TeV	172.0	$+4.4(2.6\%)$ $-5.8(3.4\%)$	$+4.7(2.7\%)$ $-4.8(2.8\%)$
LHC 8 TeV	245.8	$+6.2(2.5\%)$ $-8.4(3.4\%)$	$+6.2(2.5\%)$ $-6.4(2.6\%)$
LHC 14 TeV	953.6	$+22.7(2.4\%)$ $-33.9(3.6\%)$	$+16.2(1.7\%)$ $-17.8(1.9\%)$

Pure NNLO

Collider	σ_{tot} [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$+0.259(3.7\%)$ $-0.374(5.3\%)$	$+0.169(2.4\%)$ $-0.121(1.7\%)$
LHC 7 TeV	167.0	$+6.7(4.0\%)$ $-10.7(6.4\%)$	$+4.6(2.8\%)$ $-4.7(2.8\%)$
LHC 8 TeV	239.1	$+9.2(3.9\%)$ $-14.8(6.2\%)$	$+6.1(2.5\%)$ $-6.2(2.6\%)$
LHC 14 TeV	933.0	$+31.8(3.4\%)$ $-51.0(5.5\%)$	$+16.1(1.7\%)$ $-17.6(1.9\%)$

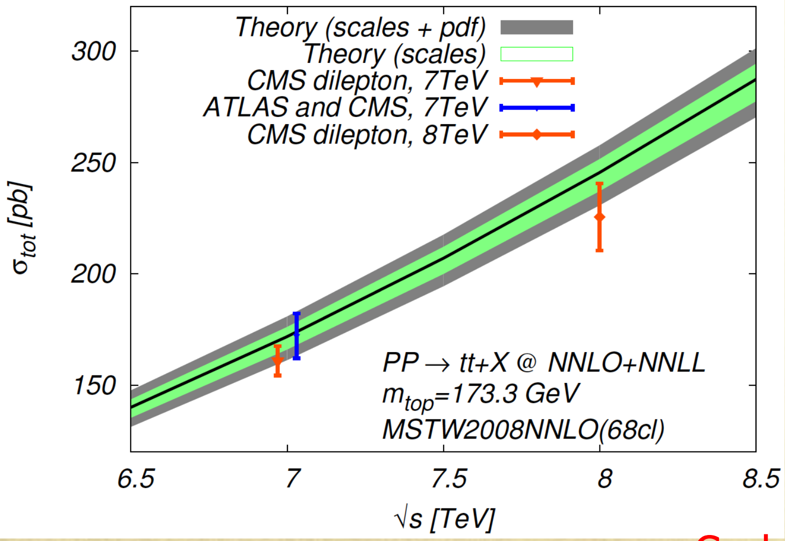
Czakon, Fiedler, Mitov '13

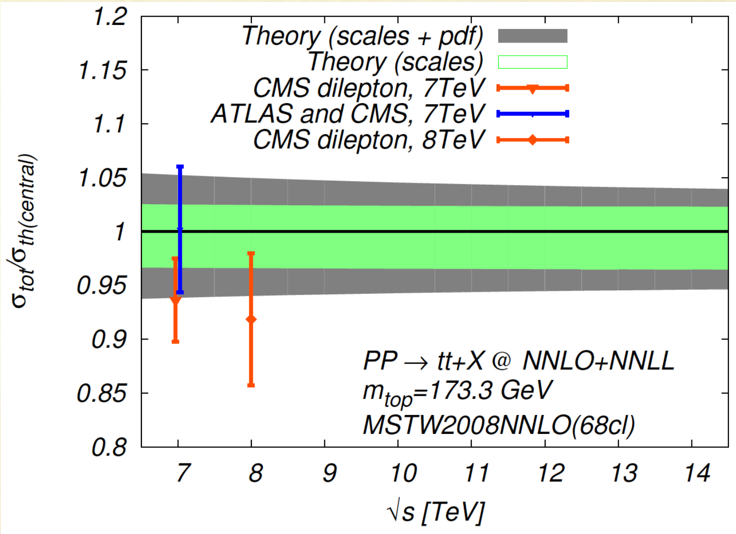
Good agreement with Tevatron measurements



- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ mt=173.3

Good agreement with LHC measurements

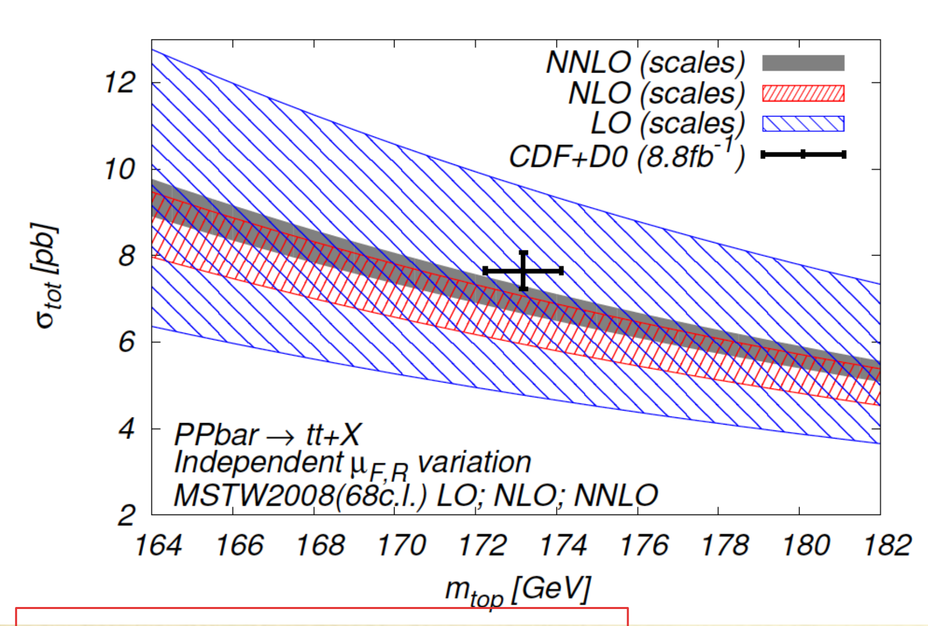




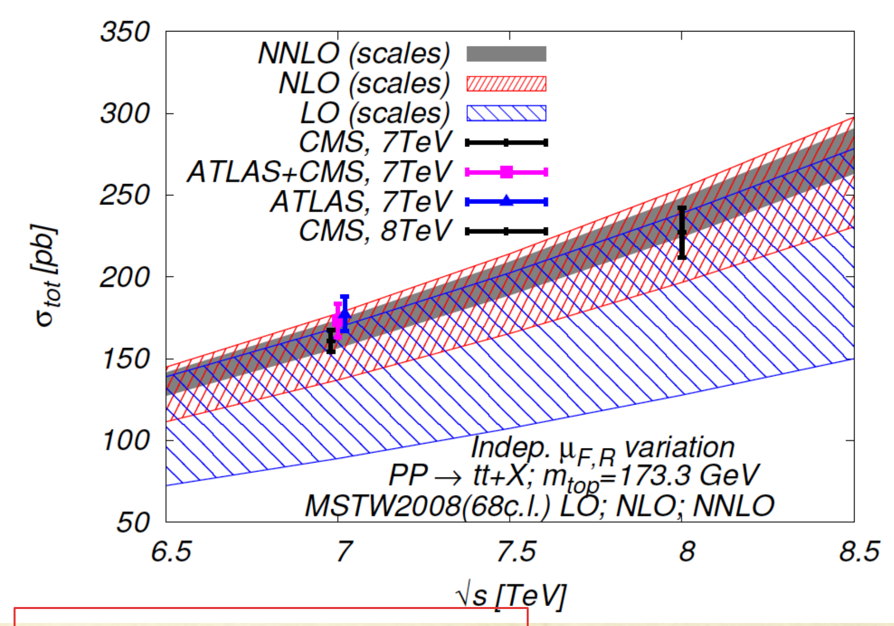
Czakon, Fiedler, Mitov '13

Good perturbative convergence

✓ Independent F/R scales variation



Scale variation @ Tevatron



Scale variation @ LHC

✓ Good overlap of various orders (LO, NLO, NNLO).

✓ Suggests the (restricted) independent scale variation is a good estimate of missing higher order terms!

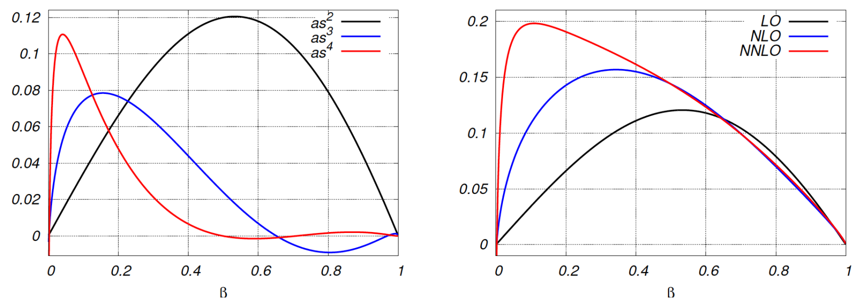
This is very important: good control over the perturbative corrections justifies less-conservative overall error estimate, i.e. more predictive theory (see next 2 slides).

For more detailed comparison, including soft-gluon resummation, see arXiv 1305.3892

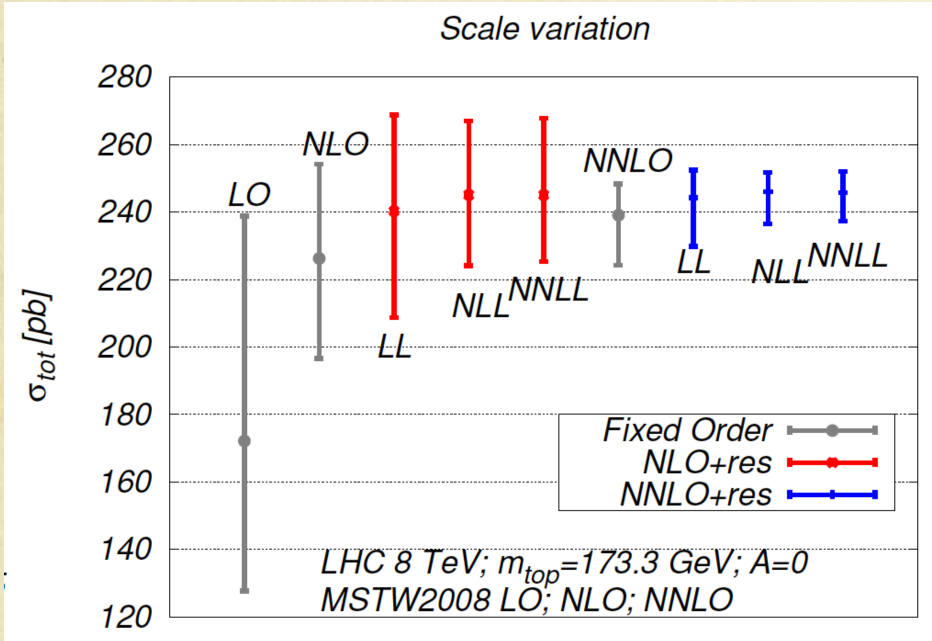
Quantifying soft-gluon resummation

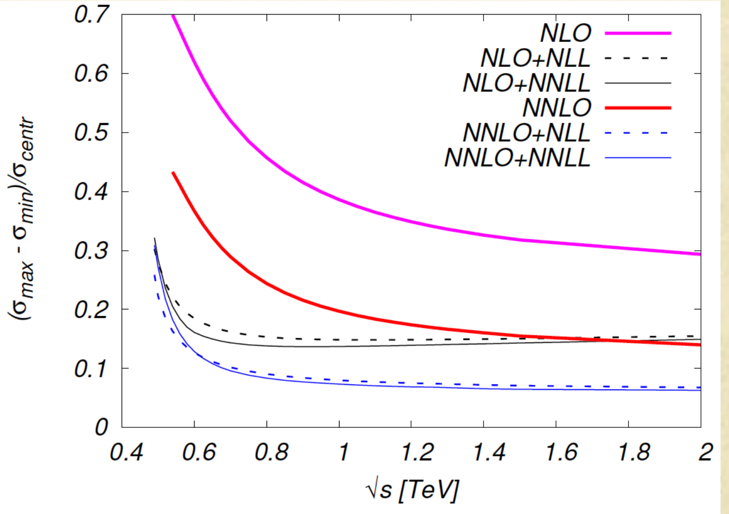
Partonic x-section's growth
close to threshold (qq reaction):

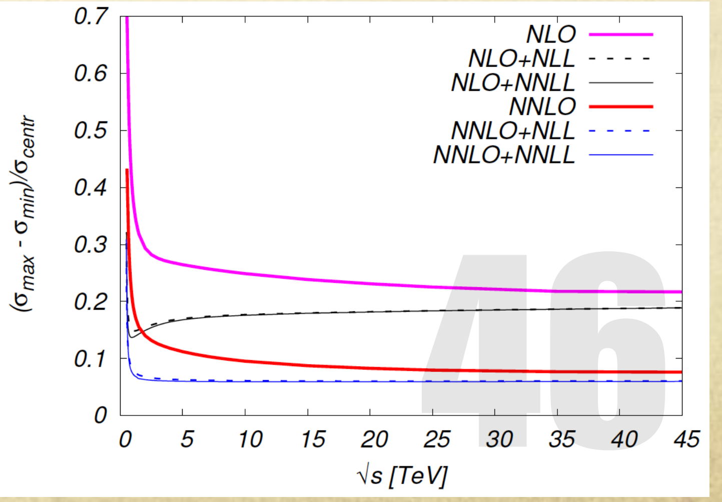
The expansion there is not converging
Resummation needed



$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \dots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \dots\right)$$







The resummed results are better, as expected.

Update of: Cacciari, Czakon, Mangano, Mitov, Nason '11

LHC: general features at NNLO+NNLL

Czakon, Fiedler, Mitov '13

Czakon, Mangano, Mitov, Rojo '13

✓ We have reached a point of saturation: uncertainties due to

- ✓ scales (i.e. missing yet-higher order corrections) ~ 3%
- ✓ pdf (at 68%cl) ~ 2-3%
- ✓ alpha_S (parametric) ~ 1.5%
- ✓ m_{top} (parametric) ~ 3%

→ All are of similar size!

✓ Soft gluon resummation makes a difference: scale uncertainty 5% → 3%

✓ The total uncertainty tends to decrease when increasing the LHC energy

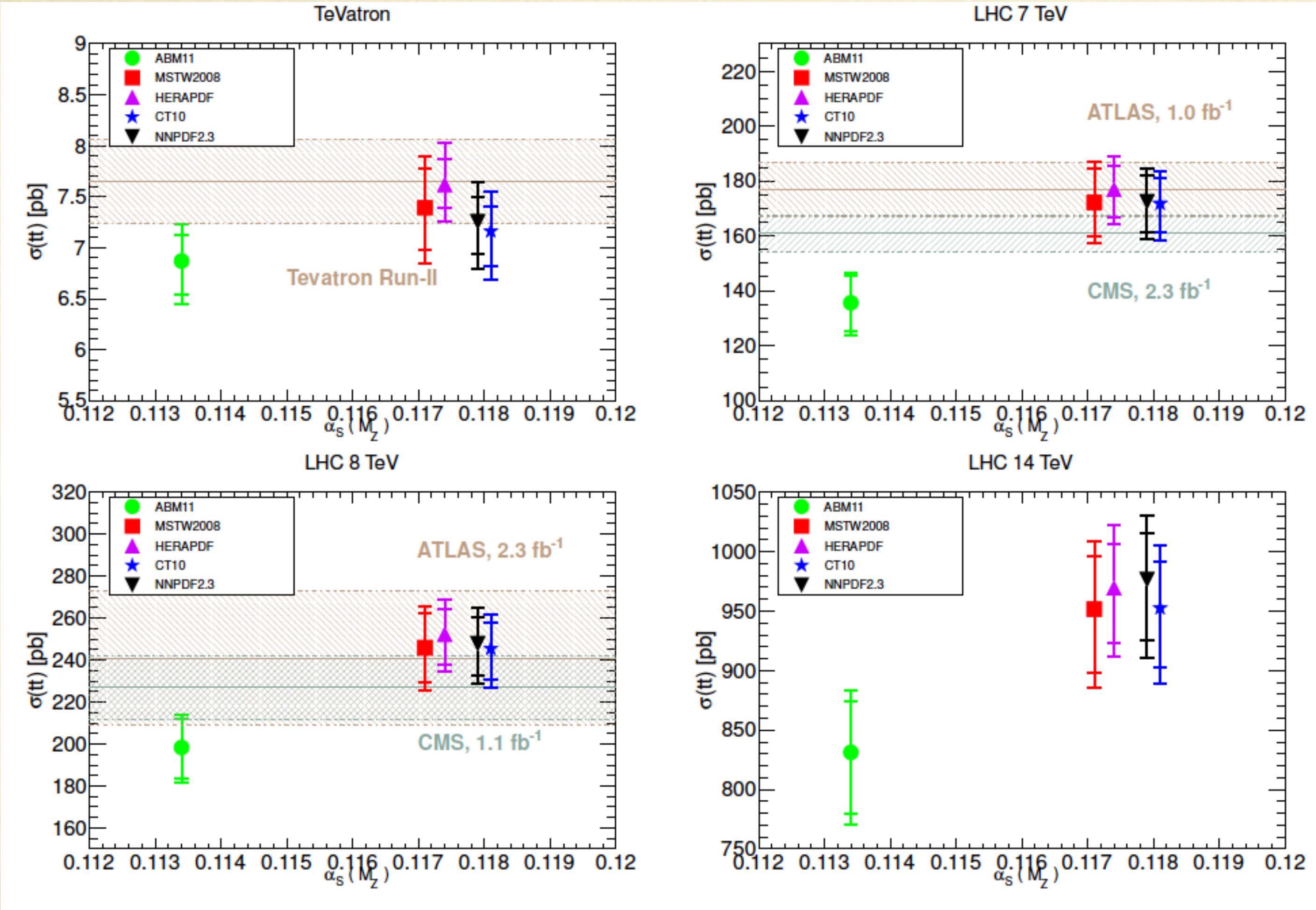
Application to PDF's

Czakon, Mangano, Mitov, Rojo '13

How existing pdf sets fare when compared to existing data?

Most conservative theory uncertainty:

Scales + pdf + as + mtop



Excellent agreement between almost all pdf sets

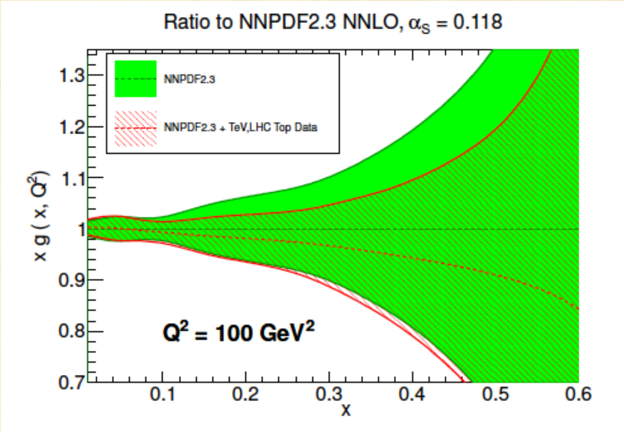
Application to PDF's

✓ tT offers for the first time a direct NNLO handle to the gluon pdf (at hadron colliders)

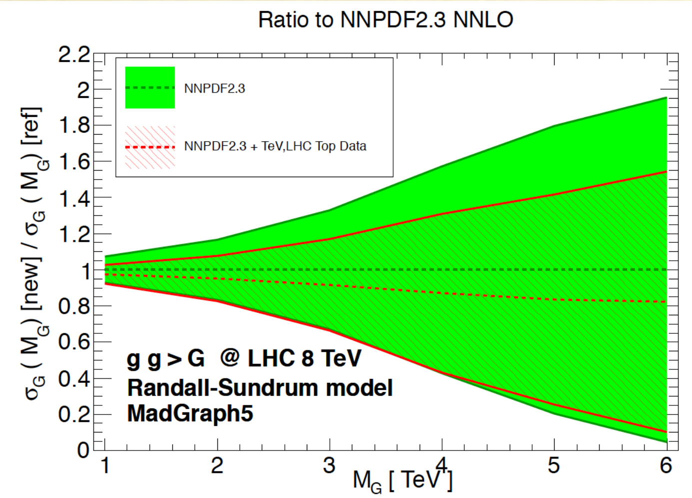
✓ implications to many processes at the LHC: Higgs and bSM production at large masses

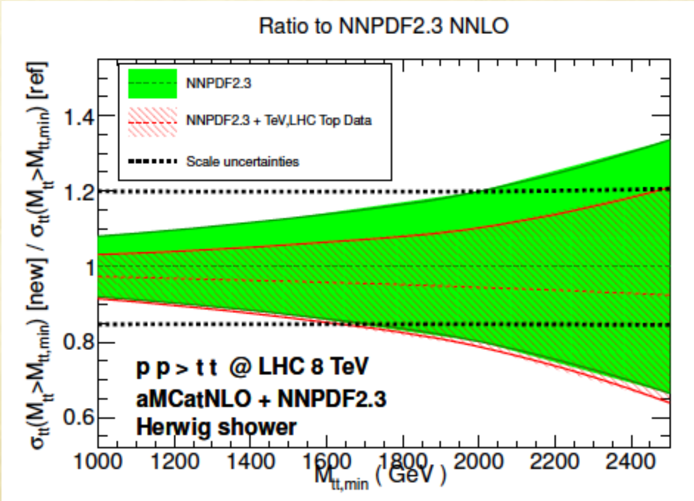
One can use the 5 available (Tevatron/LHC) data-points to improve gluon pdf

"Old" and "new" gluon pdf at large x:



… and PDF uncertainty due to "old" vs. "new" gluon pdf: Czakon, Mangano, Mitov, Rojo '13





Application to bSM searches: stealthy stop

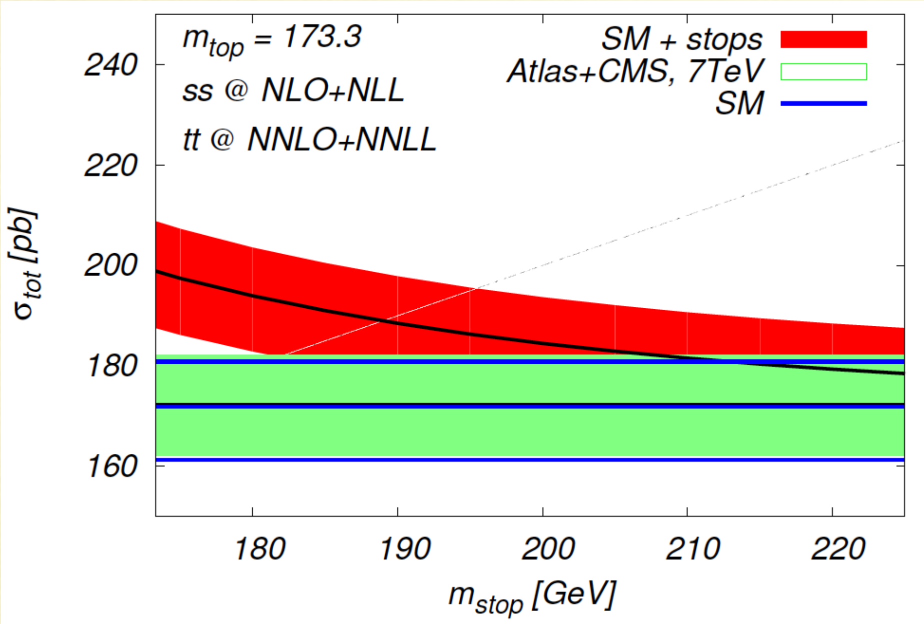
- ✓ Scenario: stop → top + missing energy
  - ✓ m_stop small: just above the top mass.
  - ✓ Stop mass < 225 GeV is allowed by current data
  - ✓ Usual wisdom: the stop signal hides in the top background

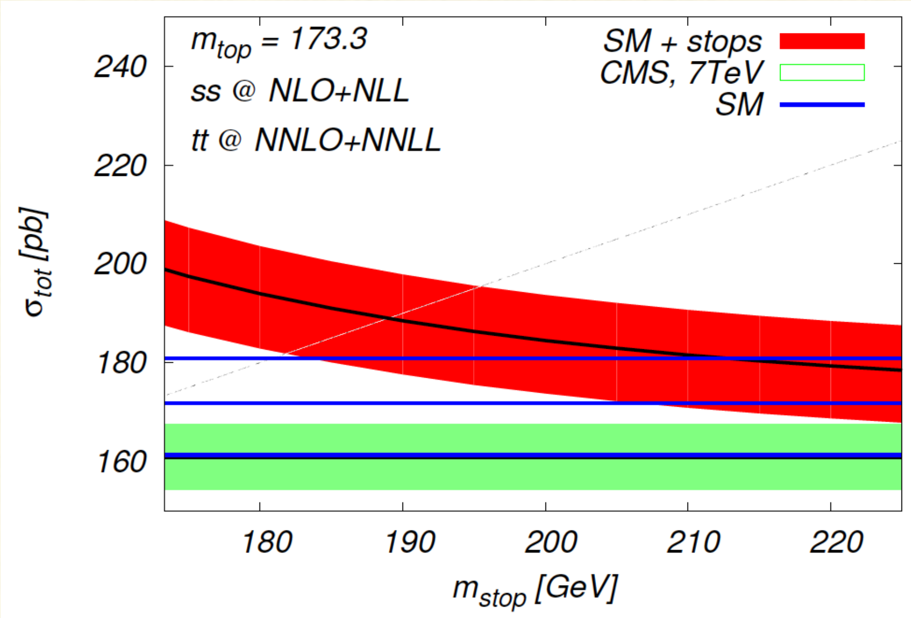
- ✓ The idea: use the top x-section to derive a bound on the stop mass. Assumptions:
  - ✓ Same experimental signature as pure tops
  - ✓ the measured x-section is a sum of top + stop
  - ✓ Use precise predictions for stop production @ NLO+NLL

    Krämer, Kulesza, van der Leeuw, Mangano, Padhi, Plehn, Portell `12
  - ✓ Total theory uncertainty: add SM and SUSY ones in quadrature.

Applications to the bSM searches: stealth stop

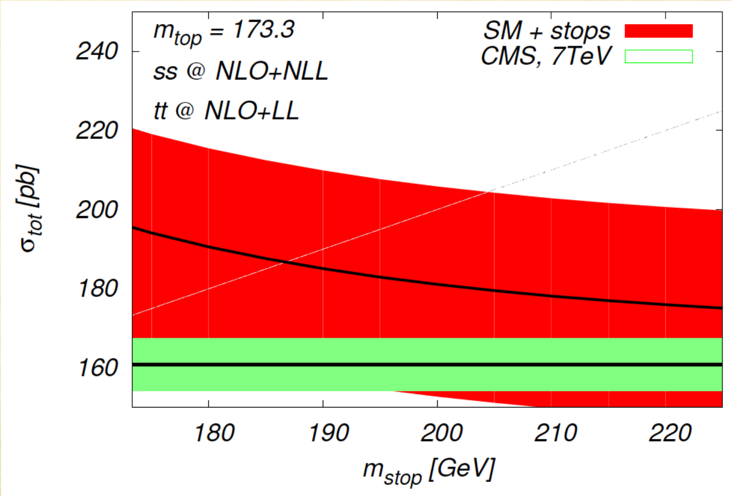
✓ Predictions Preliminary





Wonder why limits were not imposed before?

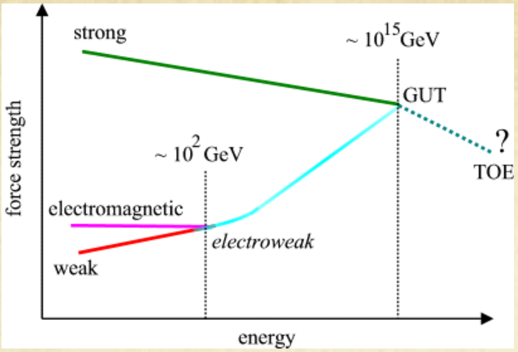
Here is the result with "NLO+shower" accuracy :



Improved NNLO accuracy makes all the difference

Where is the New Physics?

The desert …



How can we tell if it is a desert or a jungle?

Hey, top mass measurement might help!

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Top quark mass

Places where the top mass is crucial:

Bezrukov, Shaposhnikov '07-'08

- Higgs-inflation

Assume non-minimal coupling to gravity:

$$\mathcal{L}_h = -|\partial H|^2 + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 + \xi H^\dagger H \, \mathcal{R}$$

Then: Higgs = inflaton provided:

1) $10^3 < \xi < 10^4$

2) $m_h > 125.7\,\mathrm{GeV} + 3.8\,\mathrm{GeV} \left(\frac{m_t - 171\,\mathrm{GeV}}{2\,\mathrm{GeV}} \right) - 1.4\,\mathrm{GeV} \left(\frac{\alpha_s(m_Z) - 0.1176}{0.0020} \right) \pm \delta$

3) $m_h \lesssim 190\,\mathrm{GeV}$

- Theory remains perturbative at high energy,
- Has been criticized for inconsistent inflation.

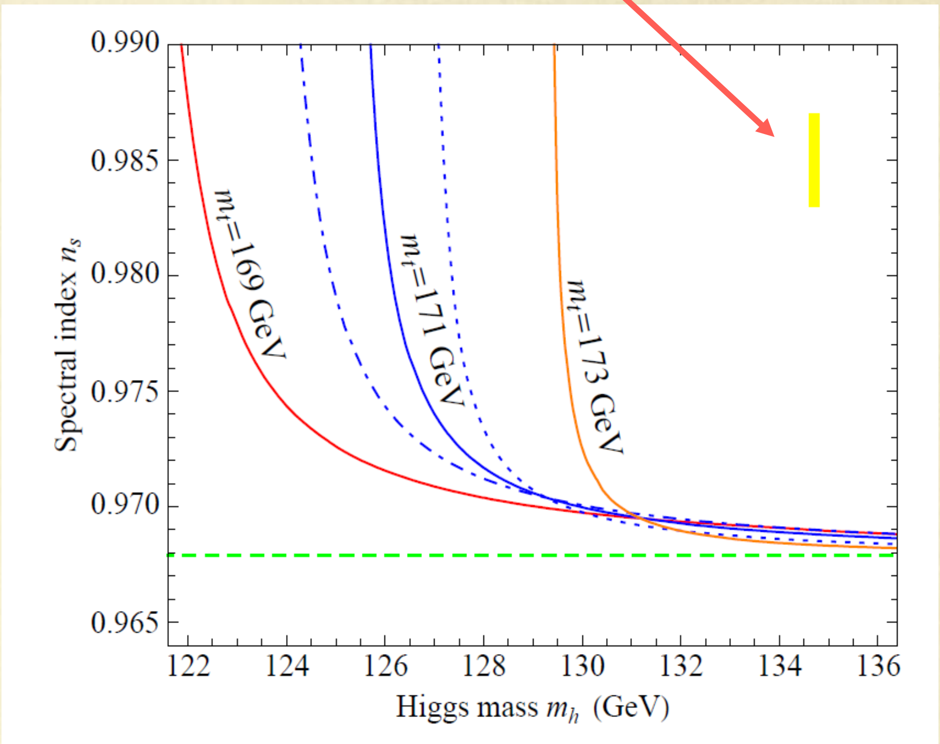
Top quark mass

Results from PLANK (past expectation – not the actual result)

- Higgs-inflation

Bezrukov, Shaposhnikov ’07-’08

Provided it works ☺
the model is very predictive!



De Simone, Hertzbergy, Wilczek arXiv:0812.4946v2

Figure 1: The spectral index n_s as a function of the Higgs mass m_h for a range of light Higgs masses. The 3 curves correspond to 3 different values of the top mass: $m_t = 169\,\mathrm{GeV}$ (red curve), $m_t = 171\,\mathrm{GeV}$ (blue curve), and $m_t = 173\,\mathrm{GeV}$ (orange curve). The solid curves are for $\alpha_s(m_Z) = 0.1176$, while for $m_t = 171\,\mathrm{GeV}$ (blue curve) we have have also indicated the 2-sigma spread in $\alpha_s(m_Z) = 0.1176 \pm 0.0020$, where the dotted (dot-dashed) curve corresponds to smaller (larger) α_s. The horizontal dashed green curve, with $n_s \simeq 0.968$, is the classical result. The yellow rectangle indicates the expected accuracy of PLANCK in measuring n_s ($\Delta n_s \approx 0.004$) and the LHC in measuring m_h ($\Delta m_h \approx 0.2\,\mathrm{GeV}$). In this plot we have set $N_e = 60$.

Yet another application of the top mass:

The fate of the Universe might depend on 1 GeV in M_{top}!

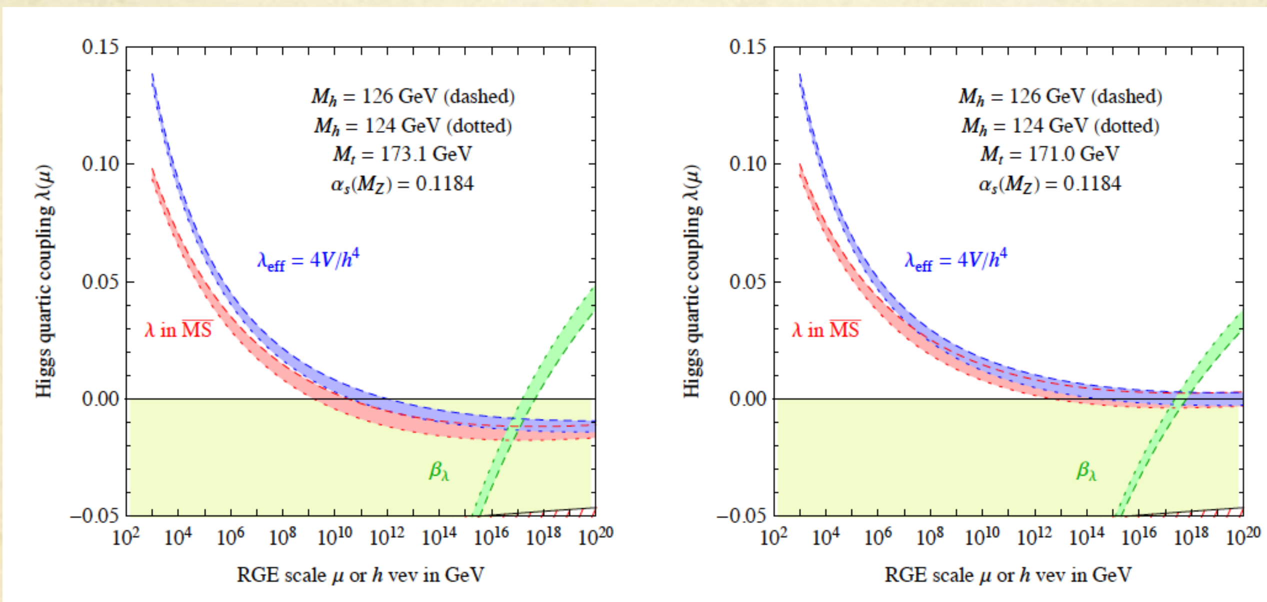
Higgs mass and vacuum stability in the Standard Model at NNLO.

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12

Vacuum stability condition:

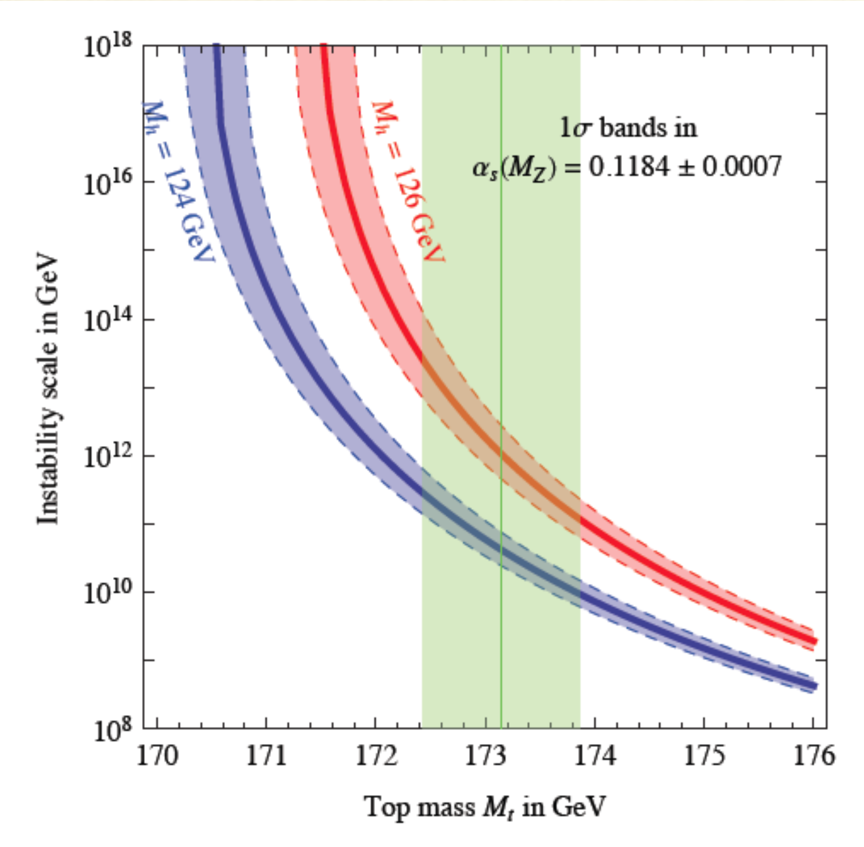
$$V_{\rm eff} = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \Delta V$$

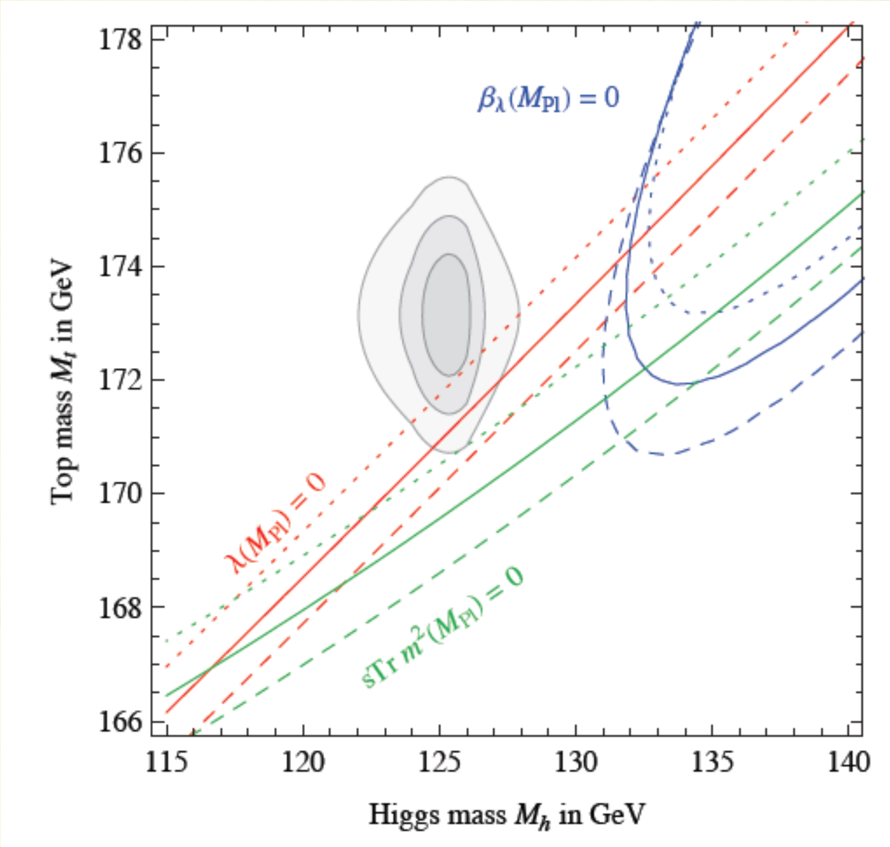
Quantum corrections (included)



Higgs mass and vacuum stability in the Standard Model at NNLO

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12





Possible implication:

For the right values of the SM parameters (and we are right there)
SM might survive the Desert.

✓ Currently a big push for better understanding of the top mass. Precision is crucial here…

Top quark mass: some thoughts

- ✓ The apparent sensitivity to m_{top} requires convincing m_{top} determination (but not for EW fits)
- ✓ What do I mean by convincing?
  - ✓ m_{top} is not an observable; cannot be measured directly.
  - ✓ It is extracted indirectly, through the sensitivity of observables to m_{top}

$$\sigma^{\rm exp}(\{Q\}) = \sigma^{\rm th}(m_t, \{Q\})$$

  - ✓ The implication: the "determined" value of m_{top} is as sensitive to theoretical modeling as it is to the measurement itself
- ✓ A worry: can there be an additional systematic O(1 GeV) shift in m_{top} ?
- ✓ The measured mass is close to the pole mass (it decays …)
- ✓ One needs to go beyond the usual MC's to achieve theoretical control
- ✓ Lots of activity (past and ongoing). A big up-to-date review:

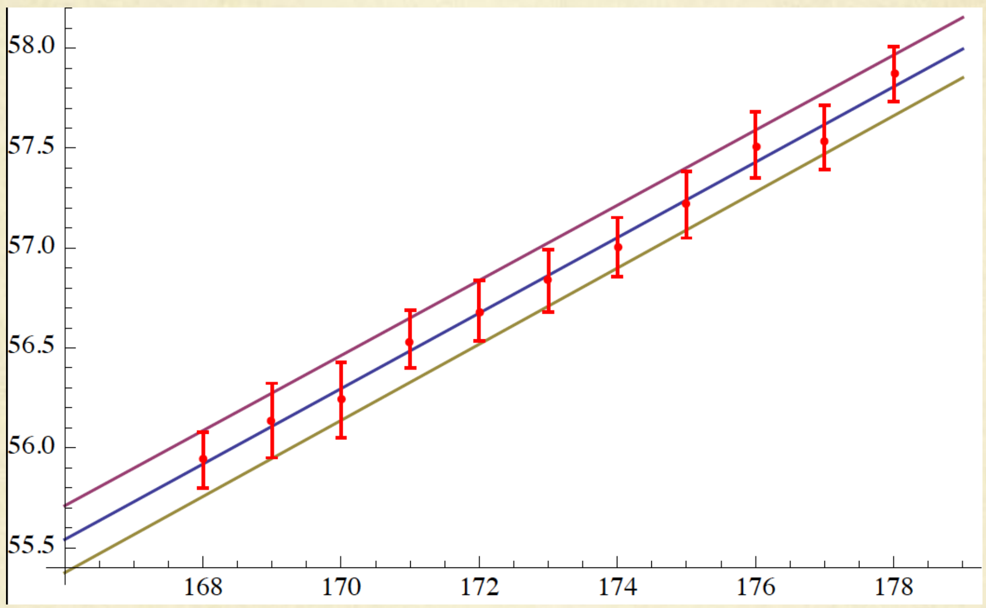
Juste, Mantry, Mitov, Penin, Skands, Varnes, Vos, Wimpenny '13

Top mass from leptonic distributions

✓ An example of an orthogonal approach (in NLO QCD)

Work with Frixione, Frederix

Measured Parameter



Predicted top mass

From this distribution, with zero exp error, we can extract m_{top} with error of 0.85 GeV

✓ One day, at NNLO, this can be improved.
✓ 8 TeV seems better than 14 TeV.

Summary and Conclusions

- Total x-section for tT production now known in full NNLO
- Result of a number of theoretical innovations
- Small scale uncertainty (2.2% Tevatron, 3% LHC). Similar to uncertainties from pdf, α_S, M_{top}
- Important phenomenology
  - Constrain and improve PDF's
  - Searches for new physics
  - Very high-precision test of SM (given exp is already at 5% !). Good agreement.

Future tasks

- This is the beginning of a new stage in precision phenomenology
  - Differential top production, with decays (NWA). A_{FB} to appear soon.
  - Any process can be computed (subject to CPU) given 2-loop amplitudes exist
  - H+1jet was already computed (expect related Z,W+jet) at NNLO

    Boughezal, Caola, Melnikov, Petriello, Schulze '13
  - Full dijet @ NNLO will become available too

    Currie, Gehrmann-De Ridder, Gehrmann, Glover, Pires '13
  - WW, etc.